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Chapter 11

LOSSES IN INDUCTION MACHINES

Losses in induction machines occur in windings, magnetic cores, besides mechanical friction and windage losses. They determine the efficiency of energy conversion in the machine and the cooling system that is required to keep the temperatures under control.

In the design stages, it is natural to try to calculate the various types of losses as precisely as possible. After the machine is manufactured, the losses have to be determined by tests. Loss segregation has become a standard method to determine the various components of losses, because such an approach does not require shaft-loading the machine. Consequently, the labor and energy costs for testings are low.

On the other hand, when prototyping or for more demanding applications, it is required to validate the design calculations and the loss segregation method. The input-output method has become standard for the scope. It is argued that, for high efficiency machines, measuring of the input and output P_{in} , P_{out} to determine losses Σp on load

$$\Sigma p = P_{in} - P_{out} \quad (11.1)$$

requires high precision measurements. This is true, as for a 90% efficiency machine a 1% error in P_{in} and P_{out} leads to a 10% error in the losses.

However, by now, less than (0.1 to 0.2)% error in power measurements is available so this objection is reduced considerably.

On the other hand, shaft-loading the IM requires a dynamometer, takes time, and energy. Still, as soon as 1912 [1] it was noticed that there was a notable difference between the total losses determined from the loss segregation method (no-load + short – circuit tests) and from direct load tests. This difference is called “stray load losses.” The dispute on the origin of “stray load losses” and how to measure them correctly is still on today after numerous attempts made so far. [2 - 8]

To reconcile such differences, standards have been proposed. Even today, only in the USA (IEEE Standard 112B) the combined loss segregation and input-output tests are used to calculate a posteriori the “stray load losses” for each motor type and thus guarantee the efficiency.

In most other standards, the “stray load losses” are assigned 0.5 or 1% of rated power despite the fact that all measurements suggest much higher values.

The use of static power converters to feed IMs for variable speed drives complicates the situation even more, as the voltage time harmonics are producing additional winding and core losses in the IM.

Faced with such a situation, we decided to retain only the components of losses which proved notable (greater than (3 to 5%) of total losses) and explore their computation one by one by analytical methods.

Further on numerical, finite element, loss calculation results are given.

11.1. LOSS CLASSIFICATIONS

The first classification of losses, based on their location in the IM, includes:

- Winding losses – stator and rotor
- Core losses – stator and rotor
- Friction & windage losses – rotor

Electromagnetic losses include only winding and core losses.

A classification of electromagnetic losses by origin would include

- Fundamental losses
 - Fundamental winding losses (in the stator and rotor)
 - Fundamental core losses (in the stator)
- Space harmonics losses
 - Space harmonics winding losses (in the rotor)
 - Space harmonic core losses (stator and rotor)
- Time harmonic losses
 - Time harmonics winding losses (stator and rotor)
 - Time harmonic core losses (stator and rotor)

Time harmonics are to be considered only when the IM is static converter fed, and thus the voltage time harmonics content depends on the type of the converter and the pulse width modulation used with it. The space harmonics in the stator (rotor) mmf and in the airgap field are related directly to mmf space harmonics, airgap permeance harmonics due to slot openings, leakage, or main path saturation.

All these harmonics produce additional (stray) core and winding losses called

- Surface core losses (mainly on the rotor)
- Tooth flux pulsation core losses (in the stator and rotor teeth)
- Tooth flux pulsation cage current losses (in the rotor cage)

Load, coil chording, and the rotor bar-tooth contact electric resistance (interbar currents) influence all these stray losses.

No-load tests include all the above components, but at zero fundamental rotor current. These components will be calculated first; then corrections will be applied to compute some components on load.

11.2. FUNDAMENTAL ELECTROMAGNETIC LOSSES

Fundamental electromagnetic losses refer to core loss due to space fundamental airgap flux density—essentially stator based—and time fundamental conductor losses in the stator and in the rotor cage or winding.

The fundamental core losses contain the hysteresis and eddy current losses in the stator teeth and core,

$$P_{FeI} \approx C_h f_1 \left[\left(\frac{B_{1ts}}{1} \right)^n G_{teeth} + \left(\frac{B_{1cs}}{1} \right)^n G_{core} \right] + C_e f_1^2 \left[\left(\frac{B_{1ts}}{1} \right)^2 G_{teeth} + \left(\frac{B_{1cs}}{1} \right)^2 G_{core} \right] \quad (11.2)$$

where C_h [W/kg], C_e [W/kg] $n = (1.7 - 2.0)$ are material coefficients for hysteresis and eddy currents dependent on the lamination hysteresis cycle shape, the electrical resistivity, and lamination thickness; G_{teeth} and G_{core} , the teeth and back core weights, and B_{1ts} , B_{1cs} the teeth and core fundamental flux density values.

At any instant in time, the flux density is different in different locations and, in some regions around tooth bottom, the flux density changes direction, that is, it becomes rotating.

Hysteresis losses are known to be different with alternative and rotating, respectively, fields. In rotating fields, hysteresis losses peak at around 1.4 to 1.6 T, while they increase steadily for alternative fields.

Moreover, the mechanical machining of stator bore (when stamping is used to produce slots) is known to increase core losses by, sometimes, 40 to 60%.

The above remarks show that the calculation of fundamental core losses is not a trivial task. Even when FEM is used to obtain the flux distribution, formulas like (11.2) are used to calculate core losses in each element, so some of the errors listed above still hold. The winding (conductor) fundamental losses are

$$P_{co} = 3R_s I_{Is}^2 + 3R_r I_{Ir}^2 \quad (11.3)$$

The stator and rotor resistances R_s and R_r are dependent on skin effect. In this sense $R_s(f_1)$ and $R_r(Sf_1)$ depend on f_1 and S . The depth of field penetration in copper $\delta_{Co}(f_1)$ is

$$\delta_{Co}(f_1) = \sqrt{\frac{2}{\mu_0 2\pi f_1 \sigma_{Co}}} = \sqrt{\frac{2}{1.256 \cdot 10^{-6} 2\pi 60 \left(\frac{f_1}{60} \right)}} = 0.94 \cdot 10^{-2} \sqrt{\frac{60}{f_1}} \text{ m} \quad (11.4)$$

If either the elementary conductor height d_{Co} is large or the fundamental frequency f_1 is large, whenever

$$\delta_{Co} > \frac{d_{Co}}{2} \quad (11.5)$$

the skin effect is to be considered. As the stator has many layers of conductors in slot even for $\delta_{Co} \approx d_{Co}/2$, there may be some skin effect (resistance increase). This phenomenon was treated in detail in Chapter 9.

In a similar way, the situation stands for the wound rotor at large values of slip S . The rotor cage is a particular case for one conductor per slot. Again,

Chapter 9 treated this phenomenon in detail. For rated slip, however, skin effect in the rotor cage is, in general, negligible.

In skewed cage rotors with uninsulated bars (cast aluminum bars), there are interbar currents (Figure 11.1a).

The treatment of various space harmonics losses at no-load follows.

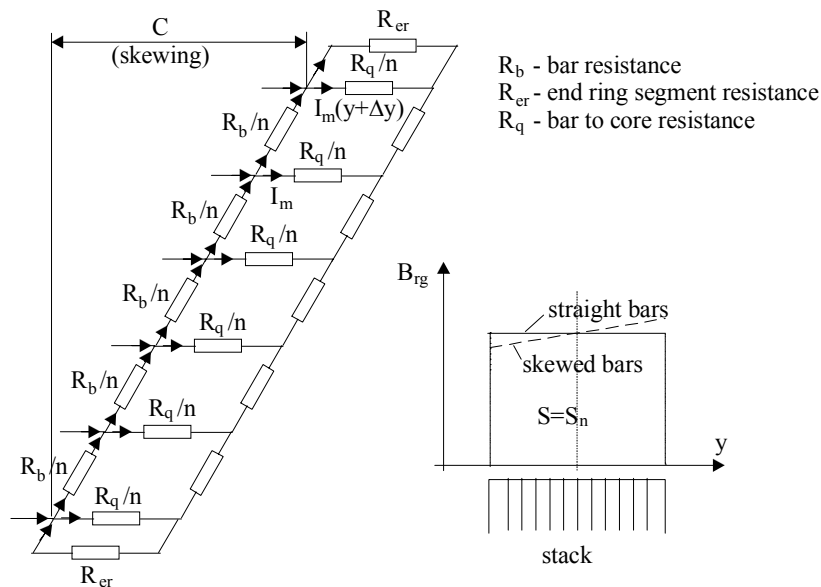


Figure 11.1 Interbar currents ($I_m(Y)$) in a skewed cage rotor a.) and the resultant airgap flux density along stack length b.)

Depending on the relative value of the transverse (contact) resistance R_q and skewing c , the influence of interbar currents on fundamental rotor conductor losses will be notable or small.

The interbar currents influence also depends on the fundamental frequency as for $f_1 = 500$ Hz, for example, the skin effect notably increases the rotor cage resistance even at rated slip.

On the other hand, skewing leaves an uncompensated rotor mmf (under load) which modifies the airgap flux density distribution along stack length (Figure 11.1b).

As the flux density squared enters the core loss formula, it is obvious that the total fundamental core loss will change due to skewing.

As skewing and interbar currents also influence the space harmonics losses, we will treat their influence once for all harmonics. The fundamental will then become a particular case.

Fundamental core losses seem impossible to segregate in a special test which would hold the right flux distribution and frequency. However a standstill

test at rated rotor frequency $f_2 = Sf_1$ would, in fact, yield the actual value of $R_r'(S_n f_1)$. The same test at various frequencies would produce precise results on conductor fundamental losses in the rotor. The stator fundamental conductor losses might be segregated from a standstill a.c. single-phase test with all phases in series at rated frequency as, in this case, the core fundamental and additional losses may be neglected by comparison.

11.3. NO-LOAD SPACE HARMONICS (STRAY NO-LOAD) LOSSES IN NONSKEWED IMs

Let us remember that airgap field space harmonics produce on no-load in nonskewed IMs the following types of losses:

- Surface core losses (rotor and stator)
- Tooth flux pulsation core losses (rotor and stator)
- Tooth flux pulsation cage losses (rotor)

The interbar currents produced by the space harmonics are negligible in nonskewed machines if the rotor end ring (bar) resistance is very small ($R_{cr}/R_b < 0.15$).

11.3.1. No-load surface core losses

As already documented in Chapter 10, dedicated to airgap field harmonics, the stator mmf space harmonics (due to the very placement of coils in slots) as well the slot openings produce airgap flux density harmonics. Further on, main flux path heavy saturation may create third flux harmonics in the airgap.

It has been shown in Chapter 10 that the mmf harmonics and the first slot opening harmonics with a number of pole pairs $v_s = N_s \pm p_1$ are attenuating and augmenting each other, respectively, in the airgap flux density.

For these mmf harmonics, the winding factor is equal to that of the fundamental. This is why they are the most important, especially in windings with chorded coils where the 5th, 7th, 11th, and 17th stator mmf harmonics are reduced considerably.

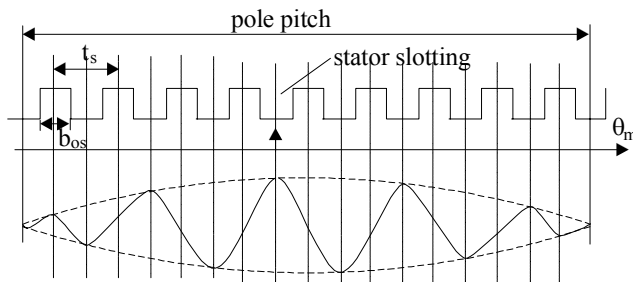


Figure 11.2 First slot opening (airgap permeance) airgap flux density harmonics

Let us now consider the fundamental stator mmf airgap field as modulated by stator slotting openings (Figure 11.2).

$$B_{N_s \pm p_1}(\theta_m, t) = \frac{\mu_0 F_{lm} a_1}{a_0} \cos(\omega_1 t - p_1 \theta_m) \cos N_s \theta_m \quad (11.6)$$

This represents two waves, one with $N_s + p_1$ pole pairs and one with $N_s - p_1$ pole pairs, that is, exactly, the first airgap magnetic conductance harmonic.

Let us consider that the rotor slot openings are small or that closed rotor slots are used. In this case, the rotor surface is flat.

The rotor laminations have a certain electrical conductivity, but they are axially insulated from each other by an adequate coating.

For frequencies of 1,200 Hz, characteristic to $N_s \pm p_1$ harmonics, the depth of field penetration in silicon steel for $\mu_{Fe} = 200 \mu_0$ is (from 11.4) $\delta \approx 0.4$ mm, for 0.6 mm thick laminations. This means that the skin effect is not significant.

Therefore we may neglect the rotor lamination-induced current reaction to the stator field. That is, the airgap field harmonics (11.6) penetrate the rotor without being disturbed by induced rotor surface eddy currents. The eddy currents along the axial direction are neglected.

But now let us consider a general harmonic of stator mmf produced field B_{vg} .

$$B_{vg} = B_v \cos\left(\frac{v\tau x_m}{p_1 \tau} - S_v \omega t\right) = B_v \cos\left(\frac{v x_m}{R} - S_v \omega t\right) \quad (11.7)$$

R —rotor radius; τ —pole pitch of the fundamental.

The slip for the v^{th} mmf harmonic, S_v , is

$$(S_v)_{S=0} = \left(1 - \frac{v}{p_1}(1 - S)\right)_{S=0} = 1 - \frac{v}{p_1} \quad (11.8)$$

For constant iron permeability μ , the field equations in the rotor iron are

$$\text{rot} B_v = 0, \quad \text{div} B_v = 0, \quad B_z = 0 \quad (11.9)$$

This leads to

$$\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} = 0 \quad \frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} = 0 \quad (11.10)$$

In iron these flux density components decrease along y (inside the rotor) direction

$$B_y = f_y(y) \cos\left(\frac{v x_m}{R} - S_v \omega t\right) \quad (11.11)$$

According to $\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0$.

$$B_x = \left(\frac{\partial f_y(y)}{\partial y} \frac{v}{R} \right) \sin \left(\frac{vx_m}{R} - S_v \omega t \right) \quad (11.12)$$

From (11.10),

$$\frac{\partial^2 f_y(y)}{\partial y^2} - \left(\frac{v}{R} \right)^2 f_y(y) = 0 \quad (11.13)$$

or
$$f_y(y) = B_v e^{\frac{vy}{R}} \quad (11.14)$$

Equation (11.14) retains one term because $y < 0$ inside the rotor and $f_y(y)$ should reach zero for $y \approx -\infty$.

The resultant flux density amplitude in the rotor iron B_{rv} is

$$B_{rv} = B_v e^{\frac{vy}{R}} \quad (11.15)$$

But the Faraday law yields

$$\text{rot} \bar{E} = \text{rot} \left(\frac{1}{\sigma_{Fe}} \bar{J} \right) = - \frac{\partial \bar{B}}{\partial t} \quad (11.16)$$

In our case, the flux density in iron has two components B_x and B_y , so

$$\begin{aligned} \frac{\partial J_y}{\partial z} &= -\sigma_{Fe} \frac{\partial B_x}{\partial t} \\ \frac{\partial J_x}{\partial z} &= -\sigma_{Fe} \frac{\partial B_y}{\partial t} \end{aligned} \quad (11.17)$$

The induced current components J_x and J_y are thus

$$\begin{aligned} J_x &= -\sigma_{Fe} S_v \omega B_v e^{\frac{vy}{R}} z \cos \left(\frac{vx_m}{R} - S_v \omega t \right) \\ J_y &= -\sigma_{Fe} S_v \omega B_v e^{\frac{vy}{R}} z \sin \left(\frac{vx_m}{R} - S_v \omega t \right) \end{aligned} \quad (11.18)$$

The resultant current density amplitude J_{rv} is

$$J_{rv} = \sqrt{J_x^2 + J_y^2} = \sigma_{Fe} S_v \omega B_v e^{\frac{vy}{R}} z \quad (11.19)$$

The losses per one lamination (thickness d_{Fe}) is

$$P_{\text{lamv}} = \frac{1}{\sigma_{\text{Fe}}} \int_{-d_{\text{Fe}}/2}^{d_{\text{Fe}}/2} \int_{y=0}^{y=-\infty} \int_{x=0}^{2\pi R} J_{\text{rv}}^2 dx dy dz \quad (11.20)$$

For the complete rotor of axial length l_{stack} ,

$$P_{0v} = \frac{\sigma_{\text{Fe}}}{24} B_v^2 (S_v \omega)^2 d_{\text{Fe}}^2 \frac{R}{v} 2\pi R l_{\text{stack}} \quad (11.21)$$

Now, if we consider the first and second airgap magnetic conductance harmonic (inversed airgap function) with $a_{1,2}$ (Chapter 10),

$$a_{1,2} = \frac{\beta}{g} F_{1,2} \left(\frac{b_{\text{os}}}{t_s} \right) \quad (11.22)$$

$\beta(b_{\text{os},r}/g)$ and $F_{1,2}(b_{\text{os},r}/t_{s,r})$ are to be found from [Table 10.1](#) and (10.14) in Chapter 10 and

$$B_v = B_{g1} \frac{a_{1,2}}{a_0}; \quad a_0 = \frac{1}{K_c g} \quad (11.23)$$

where B_{g1} is the fundamental airgap flux density with

$$v = N_s \pm p_1 \quad \text{and} \quad S_v = 1 - \frac{(N_s \pm p_1)}{p_1} \approx \frac{N_s}{p_1} \quad (11.24)$$

The no-load rotor surface losses P_{0v} are thus

$$(P_{0v})_{N_s \pm p_1} \approx 2 \frac{\sigma_{\text{Fe}}}{24} B_{g1}^2 \left(\frac{N_s}{p_1} \right)^2 (2\pi f_1)^2 d_{\text{Fe}}^2 \frac{R p_1}{N_s} 2\pi R l_{\text{stack}} \left[\frac{a_1^2 + 2a_2^2}{a_0^2} \right] \quad (11.25)$$

Example 11.1.

Let us consider an induction machine with open stator slots and $2R = 0.38$ m, $N_s = 48$ slots, $t_s = 25$ mm, $b_{\text{os}} = 14$ mm, $g = 1.2$ mm, $d_{\text{Fe}} = 0.5$ mm, $B_{g1} = 0.69$ T, $2p_1 = 4$, $n_0 = 1500$ rpm, $N_r = 72$, $t_r = 16.6$ mm, $b_{\text{or}} = 6$ mm, $\sigma_{\text{Fe}} = 10^8/45$ (Ωm)⁻¹. To determine the rotor surface losses per unit area, we first have to determine a_1 , a_2 , a_0 from (11.22). For $b_{\text{os}}/g = 14/1.2 = 11.66$ from [table 10.1](#) $\beta = 0.4$. Also, from Equation (10.14), $F_1(14/25) = 1.02$, $F_2(0.56) = 0.10$.

Also, $a_0 = \frac{1}{K_{c1,2} g}$; K_c (from Equations (5.3 – 5.5)) is $K_{c1,2} = 1.85$.

Now the rotor surface losses can be calculated from (11.25).

$$\begin{aligned} \frac{P_{0v}}{2\pi R l_{\text{stack}}} &= \frac{2 \cdot 2.22 \cdot 10^6}{24} \cdot 0.69^2 \cdot \left(\frac{48}{2} \right) (2\pi 60)^2 \cdot 0.5^2 \cdot 10^{-6} \cdot 0.19 \cdot \\ &\cdot \left[\frac{(0.4 \cdot 1.02)^2 + 2(0.4 \cdot 0.1)^2}{1.85^2} \right] = 8245.68 \text{ W/m}^2 \end{aligned}$$

As expected, with semiclosed slots, a_1 and a_2 become much smaller; also, the Carter coefficient decreases. Consequently, the rotor surface losses will be much smaller. Also, increasing the airgap has the same effect. However, at the price of larger no-load current and lower power factor of the machine. The stator surface losses produced by the rotor slotting may be calculated in a similar way by replacing $F_1(b_{os}/t_s)$, $F_2(b_{os}/t_s)$, $\beta(b_{os}/g)$ with $\beta(b_{or}/g)$, $F_1(b_{or}/t_s)$, $F_2(b_{or}/t_s)$, and N_s/p_1 with N_r/p_1 .

As the rotor slots are semiclosed, $b_{or} \ll b_{os}$, the stator surface losses are notably smaller than those of the rotor. They are, in general, neglected.

11.3.2. No-load tooth flux pulsation losses

As already documented in the previous paragraph, the stator (and rotor) slot openings produce variation in the airgap flux density distribution (Figure 11.3).

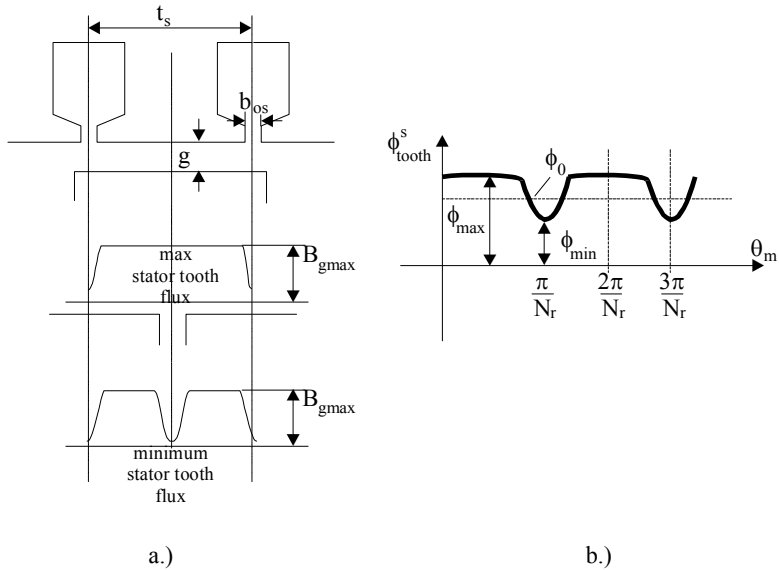


Figure 11.3 Airgap flux density as influenced by rotor and stator slotting a.) and stator tooth flux versus rotor position b.)

In essence, the total flux in a stator and rotor tooth varies with rotor position due to stator and rotor slot openings only in the case where the number of stator and rotor slots are different from each other. This is, however, the case, as $N_s \neq N_r$ at least to avoid large synchronous parasitic torques at zero speed (as demonstrated in Chapter 10).

The stator tooth flux pulsates due to rotor slot openings with the frequency $f_{PS} = N_r f \frac{(1-S)}{p_1} = N_r \frac{f}{p_1}$, for $S = 0$ (no-load). The flux variation coefficient K_ϕ

is

$$K_{\phi} = \frac{\phi_{\max} - \phi_{\min}}{2\phi_0} \quad (11.26)$$

The coefficient K_{ϕ} , as derived when the Carter coefficient was calculated in [6]

$$K_{\phi} = \frac{\gamma_2 g}{2t_s} \quad (11.27)$$

with γ_2 as

$$\gamma_2 = \frac{\left(\frac{b_{or}}{g}\right)^2}{5 + \frac{b_{or}}{g}} \quad (11.28)$$

Now, by denoting the average flux density in a stator tooth with B_{ots} , the flux density pulsation B_{p1} in the stator tooth is

$$B_{Ps} = B_{ots} \frac{\gamma_2 g}{2t_s} \quad (11.29)$$

$$B_{ots} = B_{g0} \frac{t_s}{t_s - b_{os}} \quad (11.30)$$

B_{g0} —airgap flux density fundamental.

We are now in the classical case of an iron region with an a.c. magnetic flux density B_{Ps} , at frequency $f_{Ps} = N_r f / p_1$. As $N_r f / p_1$ is a rather high frequency, eddy current losses prevail, so

$$P_{Ps0} = C_{ep} \left(\frac{B_{Ps}}{1} \right)^2 \left(\frac{f_{Ps}}{50} \right)^2 G_{steeth}; \quad G_{steeth} - \text{stator teeth weight} \quad (11.29)$$

In (11.31), C_{ep} represents the core losses at 1T and 50Hz. It could have been for 1T and 60 Hz as well.

Intuitively, the magnetic saturation of main flux path places the pulsation flux on a local hysteresis loop with lower (differential) permeability, so saturation is expected to reduce the flux pulsation in the teeth. However, Equation (11.31) proves satisfactory even in the presence of saturation. Similar tooth flux pulsation core losses occur in the rotor due to stator slotting.

Similar formulas as above are valid.

$$P_{Pr0} = C_{ep} \left(\frac{B_{Pr}}{1} \right)^2 \left(\frac{f_{Pr}}{50} \right)^2 G_{teeth}; \quad f_{pr} = N_s \frac{f}{p_1}$$

$$B_{Pr} = B_{otr} \frac{\gamma_1 g}{2t_r}; \quad B_{otr} = B_{g0} \frac{t_r}{t_r - b_{or}} \quad (11.32)$$

$$\gamma_1 = \frac{\left(\frac{b_{os}}{g}\right)^2}{5 + \frac{b_{os}}{g}}$$

As expected, C_{ep} – power losses per Kg at 1T and 50 Hz slightly changes when the frequency increases, as it does at $N_s \frac{f}{p_1}$ or $N_r \frac{f}{p_1}$. $C_{ep} = C_e K$, with $K = 1.1 - 1.2$ as an empirical coefficient.

Example 11.2

For the motor in Example 10.1, let us calculate the tooth flux pulsation losses per Kg of teeth in the stator and rotor.

Solution

In essence, we have to determine the flux density pulsations B_{Ps} , B_{Pr} , then f_{Ps} , f_{Pr} and apply Equation (11.31).

From (11.26) and (11.32),

$$\gamma_2 = \frac{\left(\frac{b_{or}}{g}\right)^2}{5 + \frac{b_{or}}{g}} = \frac{\left(\frac{6}{1.2}\right)^2}{5 + \frac{6}{1.2}} = 2.5$$

$$\gamma_1 = \frac{\left(\frac{b_{os}}{g}\right)^2}{5 + \frac{b_{os}}{g}} = \frac{\left(\frac{14}{1.2}\right)^2}{5 + \frac{14}{1.2}} = 8.166$$

$$B_{Ps} = B_{g0} \frac{t_s}{t_s - b_{or}} \frac{\gamma_2 g}{2t_s} = \frac{0.69 \cdot 2.5 \cdot 1.2}{2(25 - 14)} = 0.094T$$

$$B_{Pr} = B_{g0} \frac{t_r}{t_r - b_{or}} \frac{\gamma_1 g}{2t_r} = \frac{0.69 \cdot 8.166 \cdot 1.2}{2(16.6 - 6)} = 0.3189T$$

With $C_{ep} = 3.6$ W/Kg at 1T and 60 Hz,

$$P_{Ps} = C_{ep} \left(\frac{B_{Ps}}{1}\right)^2 \left(\frac{f_{Pr}}{50}\right)^2 = 3.6 \left(\frac{0.094}{1}\right)^2 \left(\frac{72 \cdot 60}{2 \cdot 50}\right)^2 = 59.3643W / Kg$$

$$P_{Pr} = C_{ep} \left(\frac{B_{Pr}}{1} \right)^2 \left(\frac{f_{ps}}{50} \right)^2 = 3.6 \left(\frac{0.3189}{1} \right)^2 \left(\frac{48 \cdot 60}{2 \cdot 50} \right)^2 = 303.66 \text{ W / Kg}$$

The open slots in the stator produce large rotor tooth flux pulsation no-load specific losses (P_{Pr}).

The values just obtained, even for straight rotor (and stator) slots, are too large.

Intuitively we feel that at least the rotor tooth flux pulsations will be notably reduced by the currents induced in the cage by them. At the expense of no-load circulating cage-current losses, the rotor flux pulsations are reduced.

Not so for the stator unless parallel windings are used where circulating currents would play the same role as circulating cage currents.

When such a correction is done, the value of B_{Pr} is reduced by a subunitary coefficient, [4]

$$B'_{Pr} = B_{Pr} \cdot K_{tk} \cdot \sin \left(\frac{K_t p_1}{2R} \right) = K_{dk} B_{Pr}; \quad K = \frac{N_s}{p_1} \quad (11.33)$$

$$K_{tk} = 1 - \frac{L_{mk} + L_{2dk}}{L_{mk} + L_{2dk} + L_{slot} + L_{mk} \left(\frac{1}{K_{skewk}^2} - 1 \right)} \quad (11.34)$$

where L_{mk} is the magnetizing inductance for harmonic K , L_{2dk} the differential leakage inductance of K_{th} harmonic, K_{skewk} the skewing factor for harmonic K , and L_{slot} – rotor slot leakage inductance.

$$L_{mk} \approx L_m \frac{1}{K^2} \quad (11.35)$$

$$L_{2dk} \approx K_{dlk} \cdot \tau_{2dk} \cdot L_{mk} \quad (11.36)$$

$$K_{dlk} = \tanh \left(\frac{\frac{d_{Fe}}{2\delta_{Fe}}}{\frac{d_{Fe}}{2\delta_{Fe}}} \right); \quad d_{Fe} - \text{lamination thickness};$$

$$\delta_{Fe} = \sqrt{\frac{2}{\mu_{Fediff} S_k \omega \sigma_{Fe}}} \quad (11.37)$$

where μ_{Fediff} is the the iron differential permeability for given saturation level of fundamental.

$$S_k \omega = 2\pi f_k = 2\pi \frac{N_s}{p_1} \quad (11.38)$$

$$\tau_{2dk} = \left(\frac{\frac{\pi p_1 K}{R}}{\sin\left(\frac{\pi p_1 K}{R}\right)} \right)^2 - 1 \quad (11.39)$$

$$K_{\text{skewk}} = \frac{\sin\left(\frac{K p_1 c}{R}\right)}{\frac{K p_1 c}{R}} \quad (11.40)$$

Skewing is reducing the cage circulating current reaction which increases the value of K_{rk} and, consequently, the teeth flux pulsation core losses stay undamped.

Typical values for K_{dk} —the total damping factor due to cage circulating currents—would be in the range $K_{dk} = 1$ to 0.05, depending on rotor number slots, skewing, etc.

A skewing of one stator slot pitch is said to reduce the circulating cage reaction to almost zero ($K_{dk} \approx 1$), so the rotor tooth flux pulsation losses stay high. For straight rotor slots with $K_{dk} \approx 0.1 - 0.2$, the rotor tooth flux pulsation core losses are reduced to small relative values, but still have to be checked.

11.3.3. No-load tooth flux pulsation cage losses

Now that the rotor tooth flux pulsation, as attenuated by the corresponding induced bar currents, is known, we may consider the rotor bar mesh in [Figure 11.4](#).

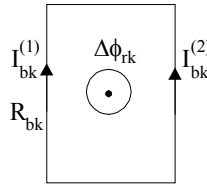


Figure 11.4 Rotor mesh with two bars

$$\Delta\phi_{rk} = K_{dk} B_{prk} (t_r - b_{or}) l_{\text{stack}} \quad (11.41)$$

In our case, $K = \frac{N_s}{p_1}$. For this harmonic, the rotor bar resistance is

increased due to skin effect by $K_{Rk} (f_k = \frac{N_s}{p_1} f_1)$.

The skin effect in the end ring is much smaller, so the ring resistance may be neglected. If we also neglect the leakage reactance in comparison with rotor bar resistance, the equation of the mesh circuit in [Figure 11.4](#) becomes

$$2\pi f K \Delta\phi_{rk} = R_b K_{Rk} (I_{bk}^{(1)} - I_{bk}^{(2)}) \quad (11.42)$$

The currents I_{bk} in the neighbouring bars (1) and (2) are phase shifted by an angle $2\pi \frac{N_s}{N_r}$, so

$$I_{bk}^{(1)} - I_{bk}^{(2)} = 2I_{bk} \sin \pi \frac{N_s}{N_r} \quad (11.43)$$

From (11.41) and (11.42),

$$I_{bk} = \frac{\Delta\phi_{rk} 2\pi f K}{2 \left(\sin \frac{\pi N_s}{N_r} \right) R_b K_{Rk}} \quad (11.44)$$

Finally, for all bars, the cage losses for harmonic K are

$$P_{ocagek} = N_r R_b K_{Rk} \left(\frac{I_{bk}}{\sqrt{2}} \right)^2 \quad (11.45)$$

or

$$P_{ocagek} \approx \frac{N_r}{8} \frac{(\Delta\phi_{rk})^2 (2\pi f)^2 \left(\frac{N_s}{p_1} \right)^2}{\sin^2 \left(\pi \frac{N_s}{N_r} \right) R_b K_{Rk}} \quad (11.46)$$

Expression (11.46) is valid for straight rotor slots. In a skewed rotor, the rotor tooth flux pulsation is reduced (per stack length) and, thus, the corresponding no-load cage losses are also reduced. They should tend to zero for one stator slot pitch skewing.

Example 11.3. Consider the motor in Example 11.1, 11.2 with the stack length $l_{stack} = 0.4$ m, rotor bar class section $A_{bar} = 250 \text{ mm}^2$, ($N_s = 48$, $N_r = 72$, $2p_1 = 4$). The rotor bar skin effect coefficient for $f_k = \frac{N_s}{p_1} f = \frac{48}{2} \cdot 60 = 1440 \text{ Hz}$ is $K_{Rk} =$

15. Let us calculate the cage losses due to rotor tooth flux pulsations if the attenuating factor of $K_{dk} = 0.05$.

Solution

With $K_{dk} = 0.05$ and $B_{pr} = 0.3189$, from Example 11.2 we may calculate $\Delta\phi_k$ (from 11.41).

$$\Delta\phi_{rk} = K_{dk} B_{prk} (t_r - b_{or})_{stack} = 0.05 \cdot 0.3189 \cdot (16.6 - 6) \cdot 10^{-3} \cdot 0.4 = 0.067 \cdot 10^{-3} \text{ Wb}$$

The d.c. bar resistance R_b is

$$R_b \approx \frac{l_{stack}}{A_{bar}} \frac{1}{\sigma_{Al}} = \frac{0.4}{250 \cdot 10^{-6}} \frac{1}{3.0 \cdot 10^7} = 0.533 \cdot 10^{-4} \Omega$$

So the bar current $(I_{bk})_{K=\frac{N_r}{p_1}}$ is (11.43)

$$I_{bk} = \frac{0.067 \cdot 10^{-3} \cdot 2\pi 60 \cdot \frac{48}{2}}{2 \sin \pi \frac{48}{72} \cdot 0.533 \cdot 10^{-4} \cdot 15} = 443 \text{ A}$$

This is indeed a large value, but let us remember that the rotor bars are many ($N_r \gg N_s$) and the stator slots are open, with a small airgap.

Now the total rotor flux pulsation losses P_{ocagek} are (11.45)

$$P_{ocagek} = 72 \cdot \left(\frac{443}{\sqrt{2}} \right)^2 \cdot 0.533 \cdot 10^{-4} \cdot 15 = 5682.2 \text{ W}$$

Let us compare this with the potential power of the IM under consideration ($f_x = 3 \cdot 10^4 \text{ N/cm}^2$ – specific force),

$$P_n \approx f_x \cdot 2\pi R \cdot L \cdot R \cdot \frac{2\pi f}{p_1} = 3 \cdot 10^4 \cdot 2\pi \cdot 0.19^2 \cdot 0.4 \cdot \frac{2\pi 50}{2} \approx 427 \text{ kW}$$

In such a case, the no-load cage losses would represent more than 1% of all power.

11.4. LOAD SPACE HARMONICS (STRAY LOAD) LOSSES IN NONSKEWED IMs

Again, slot opening and mmf space harmonics act together to produce space harmonics load (stray load) losses in the presence of larger stator and rotor currents.

In general, the no-load stray losses are augmented by load, component by component. The mmf space harmonics of the stator, for integral-slot windings, is

$$v = (6c \pm 1)p_1 \quad (11.47)$$

The slot opening (airgap magnetic conductance) harmonics are

$$v_s = c_1 N_s \pm p_1 \quad (11.48)$$

As expected, they overlap. The first slot harmonics ($c_1 = 1$) $v_{smin} = N_s \pm p_1$ are most important as their winding factor is the same as for the fundamental.

The mmf harmonic F_v amplitude is

$$F_v = F_1 \frac{K_{wv}}{K_{w1}} \frac{p_1}{v} \quad (11.49)$$

where K_{w1} and K_{wv} are the winding factors of the fundamental and of harmonic v , respectively.

If the number of slots per pole and phase (q) is not very large, the first slot (opening) harmonics $N_s \pm p_1$ produce the largest field in the airgap.

Only for full – pitch windings may the phase belt mmf harmonics ($v = 5, 7, 11, 13$) produce airgap fields worthy of consideration because their winding factors are not small enough (as they are in chorded coil windings). However, even in such cases, the losses produced by the phase belt mmf harmonics may be neglected by comparison with the first slot opening harmonics $N_s \pm p_1$. [6]

Now, for these first slot harmonics, it has been shown in Chapter 10 that their mmf companion harmonics of same order produce an increase for $N_s - p_1$ and a decrease for $N_s + p_1$, [6]

$$\xi_{(N_s - p_1)0} = \left(1 + \frac{N_s - p_1}{p_1} \frac{a_1}{2a_0} \right) \frac{1}{K_c} > 1 \quad (11.50)$$

$$\xi_{(N_s + p_1)0} = \left(1 - \frac{N_s + p_1}{p_1} \frac{a_1}{2a_0} \right) \frac{1}{K_c} < 1$$

where a_1 , a_0 , K_s have been defined earlier in this chapter for convenience.

For load conditions, the amplification factors $\xi_{(N_s - p_1)0}$ and $\xi_{(N_s + p_1)0}$ are to be replaced by [6]

$$\begin{aligned} \xi_{(N_s - p_1)n} &= \frac{1}{K_c} \sqrt{1 + 2(\sin \varphi_n) \frac{I_0}{I_n} \frac{N_s - p_1}{p_1} \frac{a_1}{2a_0} + \left(\frac{I_0}{I_n} \frac{N_s - p_1}{p_1} \frac{a_1}{2a_0} \right)^2} \\ \xi_{(N_s + p_1)n} &= \frac{1}{K_c} \sqrt{1 - 2(\sin \varphi_n) \frac{I_0}{I_n} \frac{N_s + p_1}{p_1} \frac{a_1}{2a_0} + \left(\frac{I_0}{I_n} \frac{N_s + p_1}{p_1} \frac{a_1}{2a_0} \right)^2} \end{aligned} \quad (11.51)$$

I_0 is the no-load current, I_n the current under load, and φ_n the power factor angle on load.

From now on using the above amplification factors, we will calculate the correction coefficients for load to multiply the various no-load stray losses and thus find the load stray losses.

Based on the fact that losses are proportional to harmonic flux densities squared, for $N_s \pm P_1$ slot harmonics we obtain

- Rotor surface losses on load

The ratio between load P_{0n} and no-load P_{00} surface losses C_{loads} is

$$P_{0n} = P_{00} \cdot C_{\text{loads}}; \quad C_{\text{loads}} > 1 \quad (11.52)$$

$$C_{\text{loads}} = \left(\frac{I_n}{I_0} \right)^2 \frac{\left[\left(\frac{p_1}{N_s - p_1} \right)^2 \xi^{2(N_s - p_1)n} + \left(\frac{p_1}{N_s + p_1} \right)^2 \xi^{2(N_s + p_1)n} \right]}{\left[\left(\frac{p_1}{N_s - p_1} \right)^2 \xi^{2(N_s - p_1)0} + \left(\frac{p_1}{N_s + p_1} \right)^2 \xi^{2(N_s + p_1)0} \right]} \quad (11.53)$$

- Tooth flux pulsation load core losses

For both stator and rotor, the no-load surface P_{sp0} , P_{rp0} are to be augmented by amplification coefficients similar to C_{loads} ,

$$\begin{aligned} P_{spn} &= P_{sp0} \cdot C_{\text{loadr}} \\ P_{rpn} &= P_{rp0} \cdot C_{\text{loads}} \end{aligned} \quad (11.54)$$

where C_{loadr} is calculated with N_r instead of N_s .

- Tooth flux pulsation cage losses on load

For chorded coil windings the same reasoning as above is used.

$$P_{\text{ncage}} = P_{0\text{cage}} \cdot C_{\text{loadr}} \quad (11.55)$$

For full pitch windings the 5th and 7th (phase belt) mmf harmonics losses are to be added.

The cage equivalent current I_{rv} produced by a harmonic v is

$$I_{rv} \sim \frac{F_v v}{1 + \tau_v} = F_1 \frac{K_{wv}}{K_{w1}} \frac{p_1}{1 + \tau_v} \quad (11.56)$$

τ_v is the leakage (approximately differential leakage) coefficient for harmonic v ($\tau_v \approx \tau_{dv}$).

The cage losses,

$$P_{rv} \sim I_{rv}^2 R_b K_{Rv} \quad (11.57)$$

K_{Rv} is, again, the skin effect coefficient for frequency f_v .

$$f_v = S_v f_1; \quad S_v = 1 - \frac{v}{p_1} (1 - S) \quad (11.58)$$

The differential leakage coefficient for the rotor (Chapter 6),

$$\tau_{dv} = \left(\frac{\pi v}{N_r} \right)^2 \frac{1}{\sin^2 \left(\frac{\pi v}{N_r} \right)^2} - 1 \quad (11.59)$$

Equation (11.57) may be applied both for 5th and 7th ($5p_1$, $7p_1$), phase belt, mmf harmonics and for the first slot opening harmonics $N_s \pm p_1$.

Adding these four terms for load conditions [6] yields

$$P_{\text{necage}} \approx P_{0\text{cage}} \cdot C_{\text{loadr}} \left[1 + \frac{2 \left(\frac{K_{w5}}{K_{w1}} \right)^2 \frac{(1 + \tau_{dN_s})^2}{(1 + \tau_{d6})^2} \sqrt{\frac{6p_1}{N_s}}}{\xi_{(N_s - p_1)n}^2 + \xi_{(N_s + p_1)n}^2} \right] \quad (11.60)$$

$$(1 + \tau_{d6}) = \frac{\left(\frac{6\pi p_1}{N_r} \right)^2}{\sin^2 \left(\frac{6\pi p_1}{N_r} \right)}; \quad (1 + \tau_{dN_s}) = \frac{\left(\frac{\pi N_s}{N_r} \right)^2}{\sin^2 \left(\frac{\pi N_s}{N_r} \right)} \quad (11.61)$$

Magnetic saturation may reduce the second term in (11.60) by as much as 60 to 80%. We may use (11.61) even for chorded coil windings, but, as K_{w5} is almost zero, the factor in parenthesis is almost reduced to unity.

Example 11.4. Let us calculate the stray load amplification with respect to no-load for a motor with nonskewed insulated bars and full pitch stator winding; open stator slots, $a_0 = 0.67/g$, $a_1 = 0.43/g$, $I_0/I_n = 0.4$, $\cos\phi_n = 0.86$, $N_s = 48$, $N_r = 40$, $2p_1 = 4$, $\beta = 0.41$, $K_c = 1.8$.

Solution

We have to first calculate from (11.50) the amplification factors for the airgap field $N_s \pm p_1$ harmonics by the same order mmf harmonics.

$$\xi_{(N_s - p_1)0} = \frac{1}{K_c} \left(1 + \frac{N_s - p_1}{p_1} \cdot \frac{a_1}{2a_0} \right) = \frac{1}{K_c} \left(1 + \frac{(48 - 2)}{2} \cdot \frac{0.43}{2 \cdot 0.67} \right) = \frac{8.4}{K_c}$$

$$\xi_{(N_s + p_1)0} = \frac{1}{K_c} \left(1 - \frac{N_s + p_1}{p_1} \cdot \frac{a_1}{2a_0} \right) = \frac{1}{K_c} \left(1 - \frac{(48 + 2)}{2} \cdot \frac{0.43}{2 \cdot 0.67} \right) = \frac{-7}{K_c}$$

Now, the same factors under load are found from (11.51).

$$\xi_{(N_s - p_1)n} = \frac{1}{K_c} \sqrt{1 + 2 \cdot 0.53 \cdot 0.4 \cdot \frac{48 - 2}{2} \cdot \frac{0.43}{2 \cdot 0.67} + \left(0.4 \cdot \frac{48 - 2}{2} \cdot \frac{0.43}{2 \cdot 0.67} \right)^2} = \frac{3.6}{K_c}$$

$$\xi_{(N_s + p_1)n} = \frac{1}{K_c} \sqrt{1 - 2 \cdot 0.53 \cdot 0.4 \cdot \frac{48 + 2}{2} \cdot \frac{0.43}{2 \cdot 0.67} + \left(0.4 \cdot \frac{48 + 2}{2} \cdot \frac{0.43}{2 \cdot 0.67} \right)^2} = \frac{2.8}{K_c}$$

The stray loss load amplification factor C_{loadr} is (11.53)

$$C_{\text{loads}} \approx \left(\frac{I_n}{I_0} \right)^2 \left[\frac{\xi^2 (N_s - p_1)_n + \xi^2 (N_s + p_1)_n}{\xi^2 (N_s - p_1)_0 + \xi^2 (N_s + p_1)_0} \right] = \left(\frac{1}{0.4} \right)^2 \left[\frac{3.60^2 + 2.80^2}{8.4^2 + 7^2} \right] = 1.0873!$$

So the rotor surface and the rotor tooth flux pulsation (stray) load losses are increased at full load only by 8.73% with respect to no-load.

Let us now explore what happens to the no-load rotor tooth pulsation (stray) cage losses under load. This time we have to use (11.60),

$$\frac{P_{\text{ncage}}}{P_{\text{0cage}}} = C_{\text{load}} \cdot \left[1 + \frac{2K_s \left(\frac{K_{w5}}{K_{w1}} \right)^2 \frac{(1 + \tau_{dN_s})^2}{(1 + \tau_{d6})^2} \sqrt{\frac{6p_1}{N_s}}}{\xi^2 (N_s - p_1)_n + \xi^2 (N_s + p_1)_n} \right]$$

with K_s a saturation factor ($K_s = 0.2$), $K_{w5} = 0.2$, $K_{w1} = 0.965$. Also τ_{d48} and τ_{d6} from (11.61) are $\tau_{d48} = 42$, $\tau_{d6} = 0.37$. Finally,

$$\frac{P_{\text{ncage}}}{P_{\text{0cage}}} = 1.0873 \cdot \left[1 + \frac{2 \cdot 0.2 \left(\frac{0.2}{0.965} \right)^2 \frac{(1 + 42)^2}{(1 + 0.37)^2} \sqrt{\frac{6 \cdot 2}{48}}}{3.60^2 + 2.80^2} \right] = 1.43!$$

As expected, the load stray losses in the insulated nonskewed cage are notably larger than for no-load conditions.

11.5. FLUX PULSATION (STRAY) LOSSES IN SKEWED INSULATED BARS

When the rotor slots are skewed and the rotor bars are insulated from rotor core, no interbar currents flow between neighboring bars through the iron core.

In this case, the cage no-load stray losses P_{0cage} due to first slot (opening) harmonics $N_s \pm p_1$ are corrected as [6]

$$P_{\text{0cages}} = \frac{P_{\text{0cage}}}{2} \left[\left(\frac{\sin \frac{\pi c}{t_s N_s} (N_s + p_1)}{\frac{\pi c}{t_s N_s} (N_s + p_1)} \right)^2 + \left(\frac{\sin \frac{\pi c}{t_s N_s} (N_s - p_1)}{\frac{\pi c}{t_s N_s} (N_s - p_1)} \right)^2 \right] \quad (11.62)$$

where c/t_s is the skewing in stator slot pitch t_s units.

When skewing equals one stator slot pitch ($c/t_s = 1$), Equation (11.62) becomes

$$(P_{\text{0cages}})_{c/t_s=1} \approx P_{\text{0cage}} \cdot \left(\frac{p_1}{N_s} \right)^2 \quad (11.63)$$

Consequently, in general, skewing reduces the stray losses in the rotor insulated (on no-load and on load) as a bonus from (11.60). For one stator pitch skewing, this reduction is spectacular.

Two things we have to remember here.

- When stray cage losses are almost zero, the rotor flux pulsation core losses are not attenuated and are likely to be large for skewed rotors with insulated bars.
- Insulated bars are made generally of brass or copper.

For the vast majority of small and medium power induction machines, cast aluminum uninsulated bar cages are used. Interbar current losses are expected and they tend to be augmented by skewing.

We will treat this problem separately.

11.6. INTERBAR CURRENT LOSSES IN NONINSULATED SKEWED ROTOR CAGES

For cage rotor IMs with skewed slots (bars) and noninsulated bars, transverse or cross-path or interbar additional currents occur through rotor iron core between adjacent bars.

Measurements suggest that the cross-path or transverse impedance Z_d is, in fact, a resistance R_d up to at least $f = 1$ kHz. Also, the contact resistance between the rotor bar and rotor teeth is much larger than the cross-path iron core resistance. This resistance tends to increase with the frequency of the harmonic considered and it depends on the manufacturing technology of the cast aluminum cage rotor. To have a reliable value of R_d , measurements are mandatory.

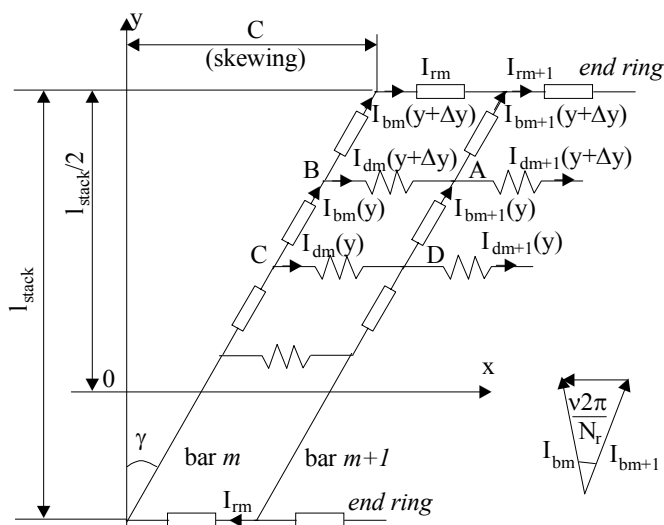


Figure 11.5 Bar and interbar currents in a rotor cage with skewed slots

To calculate the interbar currents (and losses), a few analytical procedures have been put forward [9, 6, 10]. While [9, 10] do not refer especially to the first slot opening harmonics, [6] ignores the end ring resistance.

In what follows, we present a generalization of [9, 11] to include the first slot opening harmonics and the end ring resistance.

Let us consider the rotor stack divided into many segments with the interbar currents lumped into definite resistances (Figure 11.5).

The skewing angle γ is

$$\tan \gamma = \frac{c}{l_{\text{stack}}} \quad (11.64)$$

What airgap field harmonics are likely to produce interbar currents? In principle, all space harmonics, including the fundamental, do so. Additionally, time harmonics have a similar effect. However, for chorded coil stator windings, the first slot harmonic $N_s - p_1$ is augmented by the same stator mmf harmonic; $N_s + p_1$ harmonic is attenuated in a similar way as shown in previous paragraphs.

Only for full-pitch stator windings, the losses produced by the first two 5th and 7th phase belt stator mmf harmonics are to be added. (For chorded coil windings, their winding factors, and consequently their amplitudes, are negligible).

At 50(60) Hz apparently the fundamental component losses in the cross-path resistance for skewed noninsulated bar rotor cages may be neglected. However, this is not so in high (speed) frequency IMs (above 300Hz fundamental frequency). Also, inverter fed IMs show current and flux time harmonics, so additional interbar current losses due to the space fundamental and time harmonics are to be considered. Later in this chapter, we will return to this subject.

For the time being, let us consider only one stator frequency f with a general space harmonic v with its slip S_v .

Thus, even the case of time harmonics can be dealt with one by one changing only f , but for $v = p_1$ (pole pairs), the fundamental.

The relationship between adjacent bar and cross-path resistance currents on Figure 11.5 are

$$\begin{aligned} I_{bm}(y) - I_{bm+1}(y) &= 2je^{-\frac{j\pi v}{N_r}} I_{bm}(y) \sin \frac{\pi v}{N_r} \\ I_{dm}(y) - I_{dm+1}(y) &= 2je^{-\frac{j\pi v}{N_r}} I_{dm}(y) \sin \frac{\pi v}{N_r} \\ I_{rm} - I_{rm+1} &= 2je^{-\frac{j\pi v}{N_r}} I_{rm} \sin \frac{\pi v}{N_r} \end{aligned} \quad (11.65)$$

Kirchoff's first law in node A yields

$$I_{dm}(y + \Delta y) - I_{dm+1}(y + \Delta y) \approx \frac{I_{bm}(y + \Delta y) - I_{bm+1}(y)}{\Delta y} \quad (11.66)$$

Note that the cross path currents I_{dm} , I_{dm+1} refer to unity stack length.

With (11.65) in (11.66), we obtain

$$I_{dm}(y) \approx -j \frac{e^{-\frac{j\pi v}{N_r}}}{2 \sin \frac{\pi v}{N_r}} \cdot \frac{d}{dy} I_{bm}(y) \quad (11.67)$$

Also, for the ring current I_{rm} ,

$$I_{rm} - I_{rm+1} = (I_{bm+1})_{y=\frac{l_{stack}}{2}} \quad (11.68)$$

with (11.65), (11.68) becomes

$$I_{rm} = j \frac{e^{-\frac{j\pi v}{N_r}}}{2 \sin \frac{\pi v}{N_r}} (I_{bm})_{y=\frac{l_{stack}}{2}} \quad (11.69)$$

The second Kirchoff's law along the closed path ABCD yields

$$R_d l_{stack} (I_{dm}(y + \Delta y) - I_{dm}(y)) + (R_{bv} + jS_v X_{0v} (1 + \tau_{dv})) \cdot (I_{bm}(y) - I_{bm+1}(y)) \cdot \frac{\Delta y}{l_{stack}} = E_m^v \cdot e^{-\frac{j\gamma v y}{l_{stack}}} \cdot \frac{\Delta y}{l_{stack}} \quad (11.70)$$

E_m^v represents the stator current produced induced voltage in the contour ABCD. X_{0v} represents the airgap reactance for harmonic v seen from the rotor side; τ_{dv} is the differential leakage coefficient for the rotor (11.59).

Making use of (11.67) into (11.70) yields

$$R_{de} l_{stack}^2 \frac{d^2 I_{bm}(y)}{dy^2} - Z_{be} I_{bm}(y) = E_m^v e^{-\frac{j\gamma v y}{l_{stack}}} \quad (11.71)$$

The boundary conditions at $y = \pm l_{stack}/2$ are

$$(I_{rm})_{y=\pm l_{stack}/2} \cdot Z_{er} = \mp (I_{dm})_{y=\pm l_{stack}/2} R_d l_{stack}; \quad R_{de} = \frac{R_d}{4 \sin^2 \frac{v\pi}{N_r}} \quad (11.72)$$

Z_{er} represents the impedance of end ring segment between adjacent bars.

$$\underline{Z}_{er} = R_{erv} + jX_{erv}; \quad \underline{Z}_{ere} = \frac{\underline{Z}_{er}}{4 \sin^2 \frac{v\pi}{N_r}}; \quad \underline{Z}_{be} = R_{be} + jX_{be} \quad (11.73)$$

R_{erv} , X_{erv} are the resistance and reactance of the end ring segment. $\underline{Z}_{be}$ is bar equivalent impedance.

Solving the differential Equation (11.71) with (11.72) yields [10]

$$I_{bm}(y) = \frac{-E_{mv}}{(\underline{Z}_{be} + v^2 \gamma^2 R_{de} I_{stack})} \cdot \left[\left(\cos \frac{\gamma v y}{I_{stack}} - j \sin \frac{\gamma v y}{I_{stack}} \right) - A \cosh \frac{y}{I_{stack}} \sqrt{\frac{\underline{Z}_{be}}{R_{de}}} - B \sinh \frac{y}{I_{stack}} \sqrt{\frac{\underline{Z}_{be}}{R_{de}}} \right] \quad (11.74)$$

The formula of the cross-path current $I_{dm}(y)$ is obtained from (11.67) with (11.74).

Also,

$$A = \frac{\underline{Z}_{ere} \cos\left(\frac{v\gamma}{2}\right) - v\gamma R_{de} \sin\left(\frac{v\gamma}{2}\right)}{\underline{Z}_{ere} \cosh\left(\frac{1}{2} \sqrt{\frac{\underline{Z}_{be}}{R_{de}}}\right) + R_{de} \sqrt{\frac{\underline{Z}_{be}}{R_{de}}} \sinh\left(\frac{1}{2} \sqrt{\frac{\underline{Z}_{be}}{R_{de}}}\right)} \quad (11.75)$$

$$B = -j \frac{\underline{Z}_{ere} \sin\left(\frac{v\gamma}{2}\right) - v\gamma R_{de} \cos\left(\frac{v\gamma}{2}\right)}{\underline{Z}_{ere} \sinh\left(\frac{1}{2} \sqrt{\frac{\underline{Z}_{be}}{R_{de}}}\right) + R_{de} \sqrt{\frac{\underline{Z}_{be}}{R_{de}}} \cosh\left(\frac{1}{2} \sqrt{\frac{\underline{Z}_{be}}{R_{de}}}\right)} \quad (11.76)$$

When the skewing is zero ($\gamma = 0$), $B = 0$ but A is not zero and, thus, the bar current in (11.74) still varies along y (stack length) and therefore some cross-path currents still occur. However, as the end ring impedance $\underline{Z}_{ere}$ tends to be small with respect to R_{de} , the cross-path currents (and losses) are small.

However, only for zero skewing and zero end ring impedance, the interbar current (losses) are zero (bar currents are independent of y). Also, as expected, for infinite transverse resistance ($R_{de} \sim \infty$), again the bar current does not depend on y in terms of amplitude. We may now calculate the sum of the bar losses and interbar (transverse) losses together, P_{cagev}^d .

$$P_{cagev}^d = N_r \left[\int_{-\frac{I_{stack}}{2}}^{\frac{I_{stack}}{2}} |I_{bm}(y)|^2 \frac{R_b}{I_{stack}} dy + \int_{-\frac{I_{stack}}{2}}^{\frac{I_{stack}}{2}} R_d I_{stack} |I_{dm}(y)|^2 dy + 2R_{er} |I_{rm}|^2 \right] \quad (11.77)$$

Although (11.77) looks rather cumbersome, it may be worked out rather comfortably with I_{bm} (bar current) from (11.74), I_{dm} from (11.67), and I_{rm} from (11.69).

We still need the expressions of emfs E_m^v which would refer to the entire stack length for a straight rotor bar pair.

$$E_m^v = \frac{V_{rv} \left(B_v - B_v e^{-j \frac{2v\pi}{N_r}} \right) l_{stack}}{\sqrt{2}}; \text{ (RMS value)} \quad (11.78)$$

V_{rv} is the field harmonic v speed with respect to the rotor cage and B_v the v^{th} airgap harmonic field. For the case of a chorded coil stator winding, only the first stator slot (opening) harmonics $v = N_s \pm p_1$ are to be considered.

For this case, accounting for the first airgap magnetic conductance harmonic, the value of B_v is

$$B_{N_s} = B_{g1} \frac{a_1}{2a_0}; \quad N_s = (N_s \pm p_1) \mp p_1 \quad (11.79)$$

a_1 and a_0 from (11.22 and 11.23).

The speed V_{rv} is

$$V_{rv} = R \frac{2\pi f}{p_1 60} \left(1 \pm \frac{p_1}{v} \right) \quad (11.80)$$

with $v = N_s \pm p_1 \quad V_{rv} \approx R \frac{2\pi f}{p_1 60} = V_{N_s} \quad (11.81)$

The airgap reactance for the $v = N_s \pm p_1$ harmonics, X_{0v} , is

$$X_{0N_s} \approx X_{0p} \frac{p_1}{N_s} \quad (11.82)$$

X_{0p} , the airgap reactance for the fundamental, as seen from the rotor bar,

$$X_{0p} = \frac{X_m}{\alpha^2}; \quad \alpha = \frac{4 \cdot 3 \cdot W_1^2 K_{w1}^2}{N_r} \quad (11.83)$$

where X_m is the main (airgap) reactance for the fundamental (reduced to the stator),

$$X_m = \frac{6\mu_0\omega_l}{\pi^2} \frac{W_1^2 K_{w1}^2 \tau l_{stack}}{p_1 g K_c (1 + K_s)} \quad (11.84)$$

(For the derivation of (11.83), see Chapter 5.) W_1 —turns/phase, K_{w1} —winding factor for the fundamental, K_s —saturation factor, g —airgap, p_1 —pole pairs, τ —pole pitch of stator winding.

Now that we have all data to calculate the cage losses and the transverse losses for skewed uninsulated bars, making use of a computer programming

environment like Matlab etc., the problem could be solved rather comfortably in terms of numbers.

Still we can hardly draw any design rules this way. Let us simplify the solution (11.74) by considering that the end ring impedance is zero ($Z_{er} = 0$) and that the contact (transverse) resistance is small.

- Small transverse resistance

Let us consider $R_q = R_{dl} \cdot l_{stack}$, the contact transverse resistance per unit rotor length and that R_q is so small that

$$R_q \left(\frac{v\gamma}{R} \right)^2 \ll 4 \sin^2 \left(\frac{v\pi}{N_r} \right) \frac{X_{0v}(1 + \tau_{dv})}{l_{stack}} \quad (11.85)$$

For this case in [6], the following solution has been obtained:

$$I_{bm}(y) = \frac{-jE_m^v e^{-j\frac{v\gamma y}{l_{stack}}}}{X_{0v}(1 + \tau_{dv})}; \quad I_{dm} = \frac{-jE_m^v e^{-j\frac{v\gamma y}{l_{stack}}}}{2 \sin \left(\frac{v\pi}{N_r} \right) X_{0v}(1 + \tau_{dv})} \gamma \frac{l_{stack}}{n} \frac{1}{R} \quad (11.86)$$

n —the number of stack axial segments.

The transverse losses P_{d0} are

$$P_{d0} \approx \frac{R_q}{l_{stack}} \frac{N_r^3}{4\pi^2} \left(\frac{E_m^v}{X_{0v}} \right)^2 \cdot \frac{1}{1 + \tau_{dv}} \cdot \left(\frac{c}{R} \right)^2 \quad (11.87)$$

The bar losses P_{b0} are:

$$P_{b0} = N_r R_{bc} \frac{\left(E_m^v \right)^2}{X_{0v}^2 (1 + \tau_{dv})^2} \quad (11.88)$$

Equations (11.87 and 11.88) lead to remarks such as

- For zero end ring resistance and low transverse resistance R_d , the transverse (interbar) rotor losses P_d are proportional to R_d and to the skewing squared.
- The bar losses are not influenced by skewing (11.88), so skewing is not effective in this case and it shall not be used.
- Large transverse resistance

In this case, the opposite of (11.85) is true. The final expressions of transverse and cage losses P_{d0} , P_{b0} are

$$P_{q0} \approx \frac{2\pi^2}{N_r} \frac{\left(\frac{E_m^v}{l_{stack}} \right)^2}{\left(\frac{c}{R} \right)^2} l_{stack}^3 \cdot \frac{1}{(1 + \tau_{dv})} \cdot \frac{\left(1 - \frac{1}{3} \sin^2 \left(\frac{vl_{stack}}{2R} \right) \right)}{\frac{R_q}{2}} \quad (11.89)$$

$$P_{b0} = N_r R_{be} \left(\frac{E_m^v}{X_{0v}(1 + \tau_{dv})} \right)^2 K_{skewv}^2; \quad (11.90)$$

$$K_{skewv} = \frac{\sin\left(\frac{\gamma v l_{stack}}{2R}\right)}{\frac{\gamma v l_{stack}}{2R}}$$

The situation is now different.

- The transverse losses are proportional to the third power of stack length, to the square of skewing factor, and inversely proportional to transverse resistance per unit length $R_q(\Omega m)$ ($R_d \cdot l_{stack} = R_q$).
- The rotor bar losses P_b are proportional with skewing squared and, thus, by proper skewing for the first slot (opening) harmonics $v \approx N_s$, this harmonic bar losses are practically zero; skewing is effective. Also, with long stacks, only chorded windings are practical.
- With long stacks, large transverse resistivities have to be avoided by good contact between bars and tooth walls or by insulated bars. Plotting the transverse losses by the two formulas, (11.87) and (11.89), versus transverse resistance per unit length R_q shows a maximum (Figure 11.6).

So there is a critical transverse resistance R_{q0} for which the transverse losses are maximum. Such conditions are to be avoided.

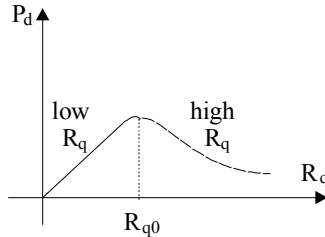


Figure 11.6 Transverse losses versus transverse resistance R_q

In [6] the critical value of R_q , R_{q0} , is

$$R_{q0} \approx \frac{X_{0v} l_{stack} 4\pi^2 \sqrt{\left[1 - \frac{1}{3} \sin^2\left(\frac{v l_{stack}}{2R}\right)\right]}}{N_r^2 \left(\frac{c}{R} v\right)^2} \quad (11.91)$$

X_{on} , R_{be} , Z_{cr} , R_d are in Ω and E_{mv} in volt.

Increasing the stack length tends to increase the critical transverse resistivity R_q . R_q is inversely proportional with skewing. Also, it seems that $(P_b)_{R_{q0}}$, or maximum transverse rotor losses, do depend on skewing.

11.7. NO-LOAD ROTOR SKEWED NONINSULATED CAGE LOSSES

For small or high transverse resistivity (R_q) expressions (11.87 – 11.90), use the computation of transverse and cage no-load stray losses with E_m^v from (11.78) and $v = N_s \pm p_1$. It has been shown that for this case, with $v \approx N_s$, the transverse losses have a minimum of $0.9 < N_r/N_s < 1.1$.

It means that for low contact (transverse) resistivity, the rotor and stator slot numbers should not be too far away from each other. Especially for $N_r/N_s < 1$, small transverse rotor losses are expected.

11.8. LOAD ROTOR SKEWED NONINSULATED CAGE LOSSES

While under no-load, the airgap permeance first harmonic was important as it acted upon the airgap flux distribution; under load, it is the stator mmf harmonics of same order $N_s \pm p_1$ that are important as the stator current increases with load. This is true for chorded pitch stator windings when the 5th and 7th pole pair phase belt harmonics are small.

The airgap flux density B_v will now be coming from a different source:

$$B_v = \frac{p_1}{v} \frac{K_{wv}}{K_{w1}} \frac{\mu_0 F_{1p}}{g K_c (1 + K_s)} \quad (11.92)$$

F_{1p} is the stator mmf fundamental amplitude.

For full pitch windings however, the 5th and 7th (phase belt) harmonics are to be considered and the conditions of low or high transverse resistivity (11.85) has to be verified for them as well with $v = 5p_1, 7p_1$.

Transverse cage losses for both smaller or higher transverse resistivity R_q are inversely proportional to differential leakage coefficient τ_{dv} which, for $v = N_s$, is

$$\tau_{dN_s} = \frac{\left(\frac{\pi N_s}{N_r} \right)^2}{\sin^2 \left(\frac{\pi N_s}{N_r} \right)} - 1 \quad (11.93)$$

It may be shown that τ_{dN_s} increases when $N_s/N_r > 1$.

Building IMs with $N_s/N_r > 1$ seems very good to reduce the transverse cage losses with skewed noninsulated rotor bars. For such designs, it may be adequate to even use non-skewed rotor slots when the additional transverse losses are almost zero.

Care must be exercised to check if the parasitic torques are small enough to secure safe starts. We should also notice that skewing leads to a small attenuation of tooth flux pulsation core losses by the rotor cage currents.

In general, the full load transverse cage loss P_{dn} is related to its value under no-load P_{d0} by the load multiplication factor C_{loads} (11.53), for chorded pitch stator windings.

$$P_{dn} = P_{d0} C_{loads} \quad (11.94)$$

For a full pitch (single layer) winding C_{loads} has to be changed to add the 5th and 7th phase belt harmonics in a similar way as in (11.60 and 11.61).

$$P_{dn} = P_{d0} C_{loads} \left[1 + \frac{2 \left(\frac{K_{w5}}{K_{w1}} \right)^2 \frac{(1 + \tau_{dN_s})}{(1 + \tau_{d6})}}{\xi_{(N_s - P_1)n}^2 + \xi_{(N_s + P_1)n}^2} \right] \quad (11.95)$$

with $\xi_{(N_s - P_1)n}$ and $\xi_{(N_s + P_1)n}$ from (11.51).

Example 11.5. For the motor with the data in Example 11.4, let us determine the P_{dn}/P_{d0} (load to no-load transverse rotor losses).

Solution

We are to use (11.95).

From Example 11.4, $\xi_{(N_s - P_1)n} = 3.6/K_c$, $K_c = 1.8$, $K_{w5} = 0.2$, $K_{w1} = 0.965$, $C_{loads} = 1.0873$, $\xi_{(N_s + P_1)n} = 2.8/K_c$, $1 + \tau_{dN_s} = 43$, $1 + \tau_{d6} = 1.37$.

We have now all data to calculate

$$\frac{P_{dn}}{P_{d0}} = 1.0873 \left[1 + \frac{2 \left(\frac{0.2}{0.965} \right)^2 \frac{43}{1.37}}{\left(\frac{3.6}{1.8} \right)^2 + \left(\frac{2.8}{1.8} \right)^2} \right] = 1.54!$$

11.9. RULES TO REDUCE FULL LOAD STRAY (SPACE HARMONICS) LOSSES

So far the rotor surface core losses, rotor and stator tooth flux pulsation core losses, and space harmonic cage losses have been included in stray load losses. They all have been calculated for motor no-load, then corrected for load conditions by adequate amplification factors.

Insulated and noninsulated, skewed and nonskewed bar rotors have been investigated. Chorded pitch and full pitch windings cause differences in terms of stray losses. Other components such as end-connection leakage flux produced

losses, which occur in the windings surroundings, have been left out as their study by analytical methods is almost impractical. In high power machines, such losses are to be considered.

Based on analysis, such as the one corroborated above, with those of [12], we line up a few rules to reduce full load stray losses

- Large number of slots per pole and phase, q , if possible, to increase the first phase belt and first slot (opening) harmonics.
- Insulated or large transverse resistance cage bars in long stack skewed rotors to reduce transverse cage losses.
- Skewing is not adequate for low transverse resistance as it does not reduce the stray cage losses while the transverse cage losses are large. Check the tooth flux pulsation core losses in skewed rotors.
- $N_r < N_s$ to reduce the differential leakage coefficient of the first slot (opening) harmonics $N_s \pm p_1$, and thus reduce the transverse cage losses.
- For $N_r < N_s$, skewing may be eliminated after the parasitic torques are checked and found small enough. For $q = 1, 2$, skewing is mandatory.
- Chorded coil windings with $y/\tau \approx 5/6$ are adequate as they reduce the first phase belt harmonics.
- With full pitch winding, use large numbers of slot/pitch/phase whenever possible.
- Skewing seems efficient for noninsulated rotor bars with high transverse resistance as it reduces the transverse rotor losses.
- With delta connection ($N_s - N_r$) $\neq 2p_1, 4p_1, 8p_1$.
- With parallel path winding, the circulating stator currents induced by the bar current mmf harmonics have to be avoided by observing a certain symmetry.
- Use small stator and rotor slot openings, if possible, to reduce the first slot opening flux density harmonics and their losses. Check the starting and peak torque as they tend to decrease due to slot leakage inductance increase.
- Use magnetic wedges for open slots, but check for the additional eddy current losses in them and secure their mechanical ruggedness.
- Increase the airgap, but maintain good power factor and efficiency.
- For one conductor per layer in slot and open slots, check that the slot opening $b_{os,r}$ to elementary conductor $d_{s,r}$ are $b_{os}/d_s \leq 1$, $b_{or}/d_r \leq 3$. This way, the stray load conductor losses by space harmonics fields are reduced.
- Use sharp tools and annealed lamination sheets (especially for low power motors), to reduce surface core losses.
- Return rotor surface to prevent laminations to be short-circuited and thus reduce rotor surface core losses.
- As storing the motor with cast aluminum (noninsulated) cage rotors after fabrication leads to a marked increase of rotor bar to slot wall contact electrical resistivity, a reduction of stray losses of (40 to 60)% may be obtained after 6 months of storage.

11.10. HIGH FREQUENCY TIME HARMONICS LOSSES

High frequency time harmonics in the supply voltage of IMs may occur either because the IM itself is fed from a PWM static power converter for variable speed or because, in the local power grid, some other power electronic devices produce voltage time harmonics at IM terminals.

For voltage-source static power converters, the time harmonics frequency content and distribution depends on the PWM strategy and the carrier ratio c_r ($2c_r$ switchings per period). For high performance symmetric regular sampled, asymmetric regular sampled, and optimal regular sampled PWM strategies, the main voltage harmonics are $(c_r \pm 2)f_1$ and $(2c_r \pm 1)f_1$, respectively, [13], [Figure 11.7](#).

It seems that accounting for time harmonics losses at frequencies close to carrier frequency suffices. As of today, the carrier ratio c_r varies from 20 to more than 200. Smaller values relate to larger powers.

Switching frequencies up to 20 kHz are typical for low power induction motors fed from IGBT voltage-source converters. Exploring the conductor and core losses up to such large frequencies becomes necessary.

High carrier frequencies tend to reduce the current harmonics and thus reduce the conductor losses associated with them, but the higher frequency flux harmonics may lead to larger core loss. On the other hand, the commutation losses in the PWM converter increase with carrier frequency.

The optimum carrier frequency depends on the motor and the PWM converter itself. The 20 kHz is typical for hard switched PWM converters. Higher frequencies are practical for soft switched (resonance) converters. We will explore, first rather qualitatively and then quantitatively, the high frequency time harmonics losses in the stator and rotor conductors and cores.

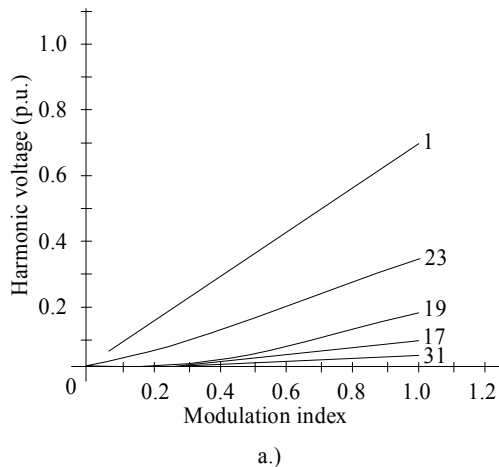


Figure 11.7 Harmonic voltages with modulation index for $c_r = 21$
a.) symmetric regular sampled PWM b.) asymmetric regular sampled PWM (continued)

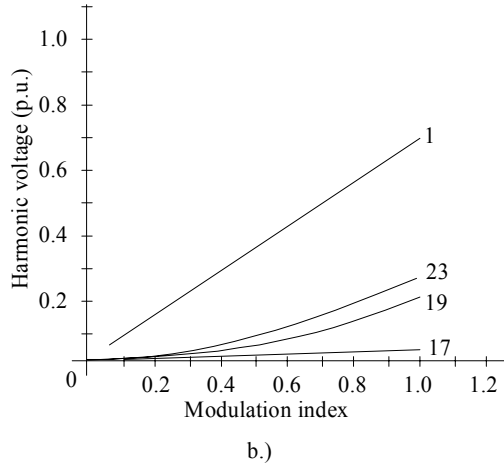


Figure 11.7 (continued)

At high carrier frequency, the skin effect, both in the conductors and iron cores, may not be neglected.

11.10.1. Conductor losses

The variation of resistance R and leakage L_1 inductance for conductors in slots with frequency, as studied in Chapter 9, is at first rapid, being proportional to f^2 . As the frequency increases further, the field penetration depth gets smaller than the conductor height and the rate of change of R and L_1 decreases to become proportional to $f^{1/2}$ (Figure 11.8).

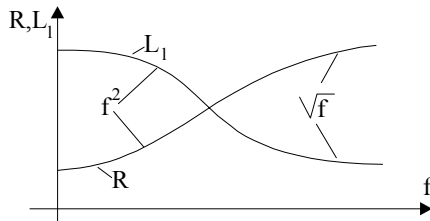


Figure 11.8 R and L_1 variation with frequency

For the end connections (or end rings), the skin effect is less pronounced, but it may be calculated with similar formulas providing virtual larger slots are defined (Chapter 9, Figure 9.10).

For high frequencies, the equivalent circuit of the IM may be simplified by eliminating the magnetization branch (Figure 11.9).

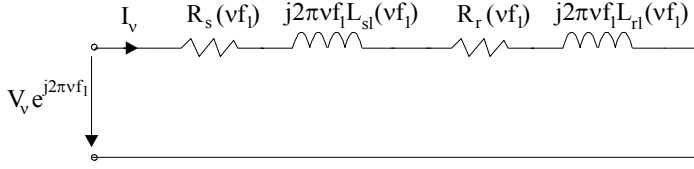


Figure 11.9 Equivalent circuit for time harmonic v

Notice that the slip $S_v \approx 1$.

In general, the reactances prevail at high frequencies,

$$I_v \approx \frac{V_v}{2\pi f_v L_\sigma(f_v)}; \quad L_\sigma(f_v) = L_{sl}(f_v) + L_{rl}(f_v) \quad (11.95)$$

The conductor losses are then

$$P_{\text{con}} = 3I_v^2(R_s(f_v) + R_r(f_v)) \approx \frac{3V_v^2}{(2\pi f_v L_\sigma(f_v))^2} (R_s(f_v) + R_r(f_v)) \quad (11.96)$$

For a given (fixed) value of the current harmonic I_v , the conductor losses will increase steadily with frequency. This case would be typical for current control.

For voltage control, when voltage harmonics V_v are given, however, (11.96) shows that it is possible to have conductor losses increasing with frequency if the decrease of leakage inductance $L_\sigma(f_v)$ with f_v and the increase factors of $(R_s(f_v) + R_r(f_v))$ with f_v are less than proportional to f_v^2 .

Measurements have shown that the leakage inductance decrease to 0.5 to 0.3 of the rated frequency (60 Hz) value at 20 kHz.

So, in general,

$$L_\sigma(f_v) \approx K_L f_v^{-0.16} \quad (11.97)$$

For high frequencies R_r variation with frequency is in the $f_v^{0.5}$ range, approximately, so the rotor conductor losses are

$$P_{\text{conf}} \approx \frac{3V_v^2}{(2\pi f_v K_L)^2 f_v^{-0.32}} K_R f_v^{0.5} \sim \frac{V_h^2}{f_v^{1.18}} \quad (11.98)$$

The rotor conductor losses drop notably as the time harmonic frequency increases.

The situation in the stator is different as there are many conductors in every slot (at least in small power induction machines).

So the skin effect for R_s will remain f_v dependent in the initial stages ($K_{Rs} = C_{Rs} f_v$),

$$P_{\text{cons}} \sim V_v^2 f_v^{0.32} \quad (11.99)$$

The stator conductor losses tend to increase slightly with frequency. For large power IMs (MW range) the stator conductor skin effect is stronger and the situation comes closer to that of the rotor: the stator conductor losses slightly decrease at higher frequencies. We should also mention that the carrier frequency in large power is only 1 to 3 kHz.

For low and medium power motors, as the carrier frequency reaches high levels (20 kHz or more), the skin effect in the stator conductors enters the $f_v^{0.5}$ domain and the stator conductor losses, for given harmonic voltage, behave like the rotor cage losses (decrease slightly with frequency (11.98)). This situation occurs when the penetration depth becomes smaller than conductor height.

11.10.2. Core losses

Predicting the core loss at high frequencies is difficult because the flux penetration depth in lamination becomes comparable with (or smaller than) the lamination thickness.

The leakage flux paths may then prevail and thus the reaction of core eddy currents may set up significant reaction fields.

The field penetration depth in laminations δ_{Fe} is

$$\delta_{Fe} = \sqrt{\frac{1}{\pi f_v \sigma_{Fe} \mu_{Fe}}} \quad (11.100)$$

σ_{Fe} —iron electrical conductivity; μ_{Fe} —iron magnetic permeability.

For $f = 60$ Hz, $\sigma_{Fe} = 2 \cdot 10^6 (\Omega m)^{-1}$, $\mu_{Fe} = 800 \mu_0$, $\delta_{Fe} = 1.63$ mm, while at 20 kHz $\delta_{Fe} = 0.062$ mm. In contrast for copper ($\delta_{Co} = 9.31$ mm, $(\delta_{Co})_{20kHz} = 0.51$ mm and in aluminum ($\delta_{Al} = 13.4$ mm, $(\delta_{Al})_{20kHz} = 0.73$ mm).

The penetration depth at $800 \mu_0$ and 20 kHz, $\delta_{Fe} = 0.062$ mm, shows the importance of skin effect in laminations. To explore the dependence of core losses on frequency, let us distinguish three cases.

- Case 1 – No lamination skin effect: $\delta_{Fe} \gg d$

This case corresponds to low frequency time harmonics. Both hysteresis and eddy current losses are to be considered.

$$P_{Fe} = (K_{hl} B_v^n f_v + K_{el} B_v^2 f_v^2) A_l l \quad (11.101)$$

where B_v is the harmonic flux density:

$$B_v = \frac{\phi_v}{A_l} \approx \frac{V_v}{2\pi f_v A_l} \quad (11.102)$$

A_l is the effective area of the leakage flux path and l is its length.

With (11.102), (11.101) becomes

$$P_{Fe} = (K_{hl} V_v^n f_v^{1-n} + K_{el} V_v^2) A_l l \quad (11.103)$$

Since $n > 1$, the hysteresis losses decrease with frequency while the eddy current losses stay constant. P_{Fe} is almost constant in these conditions.

- Case 2 – Slight lamination skin effect, $\delta_{Fe} \approx d$

When $\delta_{Fe} \approx d$, the frequency f_v is already high and thus $\delta_{Al} < d_{Al}$ and a severe skin effect in the rotor slot occurs. Consequently, the rotor leakage flux is concentrated close to the rotor surface. The “volume” where the core losses occur in the rotor decreases. In general then, the core losses tend to decrease slowly and level out at high frequencies.

- Case 3 – Strong lamination skin effect, $\delta_{Fe} < d$

With large enough frequencies, the lamination skin depth $\delta_{Fe} < d$ and thus the magnetic field is confined to a skin depth layer around the stator slot walls and on the rotor surface (Figure 11.10).

The conventional picture of rotor leakage flux paths around the rotor slot bottom is not valid in this case.

The area of leakage flux is now, for the stator, $A_l = l_f \delta_{Fe}$, with l_f the length of the meander zone around the stator slot.

$$l_f = (2h_s + b_s)N_s \quad (11.104)$$

Now the flux density B_v is

$$B_v = \frac{K_v V_v}{f_v l_f \delta_{Fe}} \sim K_f f_v^{-\frac{1}{2}} \quad (11.105)$$

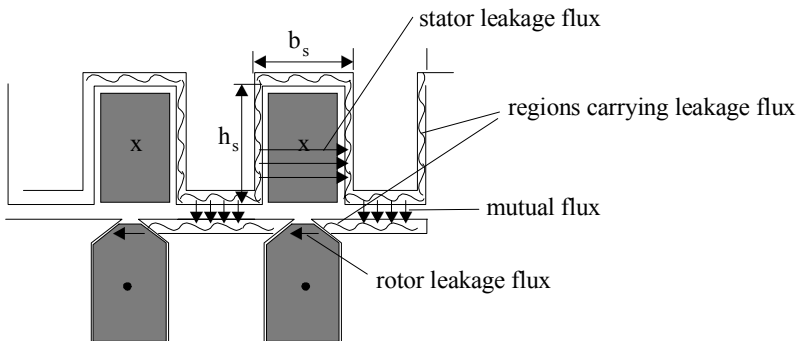


Figure 11.10 Leakage flux paths at high frequency

Consequently, the core losses (with hysteresis losses neglected) are

$$P_{Fe} = K_e B_v^2 f_v^2 l_f l_{stack} \delta_{Fe} \sim K_l V_v^2 f_v^{\frac{1}{2}} \quad (11.106)$$

A slow steady state growth of core losses at high frequencies is thus expected.

11.10.3. Total time harmonics losses

As the discussion above indicates, for a given harmonic voltage, above certain frequency, the conductor losses tend to decrease as $f_v^{-1.2}$ (11.98) while the core losses (11.106) increase.

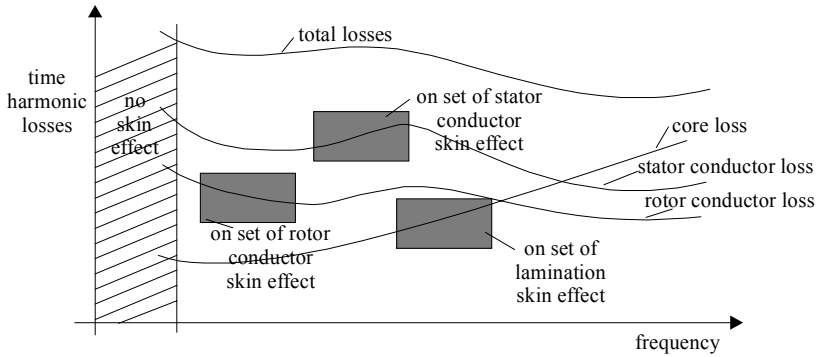


Figure 11.11 Time harmonic IM losses with frequency (constant time harmonic voltage)

Consequently, the total time harmonics may level out above a certain frequency (Figure 11.11). Even a minimum may be observed. At what frequency such a minimum occurs depends on the machine design, power, and PWM frequency spectrum.

11.11. COMPUTATION OF TIME HARMONICS CONDUCTOR LOSSES

As Equation (11.96) shows, to compute the time harmonics conductor losses, the variation of stator and rotor leakage inductances and resistances with frequency due to skin effect is needed. This problem has been treated in detail, for a single, unassigned, frequency in Chapter 9, both for the stator and rotor.

Here we simplify the correction factors in Chapter 9 to make them easier to use and interpret.

In essence, with different skin effect in the slot and end-connection (end ring) zones, the stator and resistance $R_s(f_v)$ is

$$R_s(f_v) = R_{sdc} \left[K_{Rss}(f_v) \frac{l_{stack}}{l_{coil}} + K_{Rse}(f_v) \frac{l_{endcon}}{l_{coil}} \right] \quad (11.107)$$

l_{coil} —coil length, l_{endcon} —coil end connection length.

$K_{Rss} > 1$ is the resistance skin effect correction coefficient for the slot zone and K_{Rse} corresponds to the end connection. A similar expression is valid for the stator phase leakage inductance.

$$L_{sl}(f_v) = L_{slc} \left[K_{Xlss}(f_v) \frac{l_{stack}}{l_{coil}} + K_{Xlse}(f_v) \frac{l_{endcon}}{l_{coil}} \right] \quad (11.108)$$

K_{Xlss} and K_{Xlse} are leakage inductance correction factors.

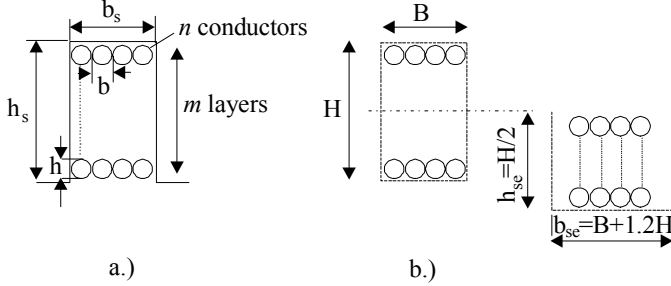


Figure 11.12 Stator slots a.) and end connection b.) geometries

Now with m layers of conductors per slot from Chapter 9, Equation (9.6),

$$K_{Rss} = \varphi(\xi) + \frac{m^2 - 1}{3} \psi(\xi) > 1 \quad (11.109)$$

$$\xi = \beta_n h, \quad \beta_n = \sqrt{\pi f_v \mu_0 \sigma_{Co} \frac{nb}{b_s}}$$

h – conductor height, n – conductors per layer, b – conductor width (or diameter), b_s – slot width. Equation (11.109) is strictly valid for rectangular slots (Figure 11.12). More evolved methods, as described in Chapter 9, may be used for general shape slots.

The same formula (11.109) may be used for end connections coefficient K_{Rse} but with b_{se} instead of b_s , h_{se} instead of h_s , and $m/2$ instead of m .

In a similar way, the reactance correction coefficient K_{Xss} is (Equation (9.7))

$$K_{Xss} = \frac{\varphi'(\xi) + \frac{(m^2 - 1)}{m^2} \psi'(\xi)}{m^2} < 1 \quad (11.110)$$

Also, from (9.4) and (9.8), (9.9)

$$\varphi(\xi) = \xi \frac{(\sinh(2\xi) + \sin(2\xi))}{(\cosh(2\xi) - \cos(2\xi))}; \quad \psi(\xi) = 2\xi \frac{(\sinh(\xi) - \sin(\xi))}{(\cosh(\xi) + \cos(\xi))} \quad (11.111)$$

$$\varphi'(\xi) = \frac{3}{2\xi} \frac{(\sinh(2\xi) - \sin(2\xi))}{(\cosh(2\xi) - \cos(2\xi))}; \quad \psi'(\xi) = \frac{(\sinh(\xi) + \sin(\xi))}{\xi(\cosh(\xi) + \cos(\xi))} \quad (11.112)$$

As the number of layers in the virtual slot of end connection is $m/2$ and the slot width is much larger than for the actual slot $b_{se} - b_s \approx 1.2h_s \approx (4 - 6)b_s$, the

skin effect coefficients for the end connection zone are smaller than for the slot zone but still, at high frequencies, they are to be considered.

For the cage rotor resistance R_r and leakage inductance L_{rl} , expressions similar to (11.107 – 11.108) are valid.

$$R_r(f_v) = R_{bdc} K_{Rb}(f_v) + R_{erdc} K_{Re}(f_v) \quad (11.113)$$

$$L_{rl}(f_v) = L_{bdc} K_{Xb}(f_v) + L_{erdc} K_{Xe}(f_v) \quad (11.114)$$

Again, expressions (11.109) and (11.110) are valid, but with $m = 1$, with $n_b = b_s$, and σ_{Al} in (11.109).

Also, in (11.110), ξ is to be calculated with $h_{se} = h$, $b_{se} = b_s + K_h h$. In other words, the end ring has been assimilated to an end connection, but the factor K_h should be 1.2 for a distant ring and less than that for an end ring placed close to the rotor stack.

Example 11.6. Time harmonics conductor losses. An 11 kW, 6 pole, $f_1 = 50$ Hz, 220 V induction motor is fed from a PWM converter at carrier frequency $f_v = 20$ kHz. For this frequency, the voltage $V_v = 126$ V (see [14]).

For the rotor: $\xi = 18.38$, $K_{Xb} = 0.0816$, $K_{Rb} = 18.38$, $R_r'(f_v) = 2.2 \Omega$ (end ring resistance neglected), $X_{rl}' = 2\pi f_v \cdot L_{rl}(f_v) = 47.9 \Omega$, and for the stator: $\xi = 3.38$, $K_{Rss} = K_{Rse} = 79.4$, $R_s(f_v) = 11.9 \Omega$, $K_{Xss} = K_{Xse} = 0.3$, $X_{sl} = 2\pi f_v \cdot L_{rl}(f_v) = 63.3 \Omega$

The harmonic current I_v is

$$I_v = \frac{V_v}{\sqrt{(R_r' + R_s)^2 + (X_{rl}' + X_{sl})^2}} = \frac{126}{\sqrt{(2.2 + 11.9)^2 + (47.9 + 63.3)^2}} \approx 1.12 \text{ A}$$

Now the conductor losses at 20 kHz are

$$(P_{con})_{20\text{kHz}} = 3(R_r' + R_s)I_v^2 = 3(2.2 + 11.9)(1.12)^2 = 53.06 \text{ W}$$

We considered above $V_v \approx V_1$, which is not the case, in general, although in some PWM strategies at some modulation factor values, such a situation may occur.

11.12. TIME HARMONICS INTERBAR ROTOR CURRENT LOSSES

Earlier in this chapter we dealt with space harmonic induced losses in skewed noninsulated bar rotor cages. This phenomenon occurs also due to time harmonics. At the standard fundamental frequency (50 or 60 Hz) the additional transverse rotor losses due to interbar currents are negligible. In high frequency motors ($f_1 > 300$ Hz), these losses count. [11] Also, time harmonics are likely to produce transverse losses for skewed noninsulated bar rotor cages.

To calculate such losses, we may use the theory developed in Paragraph 11.6, with $S = 1$, $v = 1$ but for given frequency f_v and voltage V_v .

However, to facilitate the computation process, we adopt the final results of Reference [11] which calculates the rotor resistance and leakage reactance including the skin effect and the transverse (interbar currents) losses,

$$\begin{aligned} R_r^*(f_v) = & (R_{bdc} K_{Rb}(f_v) + R_{erdc} K_{Re}(f_v)) \cdot |\alpha_0| \cos \gamma_0 + \\ & + 2\pi f_v (L_{bdc} K_{Xb}(f_v) + L_{erdc} K_{XRe}(f_v)) \cdot |\alpha_0| \sin \gamma_0 + X_m(f_v) \cdot \frac{\sin \gamma_0}{\eta^2 \left| \dot{K} \right|} \end{aligned} \quad (11.115)$$

$$\begin{aligned} X_{rl}^*(f_v) = & 2\pi f_v (L_{bdc} K_{Xb}(f_v) + L_{erdc} K_{XRe}(f_v)) \cdot |\alpha_0| \cos \gamma_0 - \\ & - (R_{bdc} K_{Rb}(f_v) + R_{erdc} K_{Re}(f_v)) \cdot |\alpha_0| \sin \gamma_0 + X_m(f_v) \cdot \left(\frac{\cos \gamma_0}{\eta^2 \left| \dot{K} \right|} - 1 \right) \end{aligned} \quad (11.116)$$

with

$$\alpha_0 = \frac{4 \cdot 3 \cdot 2\pi f_v K_{wl}^2}{N_r \cdot \dot{K}}; \quad \dot{K} = \left| \dot{K} \right| e^{j\gamma_0}; \quad \eta = \frac{\sin\left(\frac{\pi}{N_r}\right)}{\frac{\pi}{N_r}} \quad (11.117)$$

$$\begin{aligned} \dot{K} = & \frac{Z_{be} + R_{ere}}{Z_{be} + R_{de} \cdot (\gamma^2)} \cdot \left[1 - (A + B) \frac{\sinh \frac{1}{2} \left(\sqrt{\frac{Z_{be}}{R_{de}}} + j\gamma \right)}{\left(\sqrt{\frac{Z_{be}}{R_{de}}} + j\gamma \right)} - \right. \\ & \left. - (A - B) \frac{\sinh \frac{1}{2} \left(\sqrt{\frac{Z_{be}}{R_{de}}} + j\gamma \right)}{\left(\sqrt{\frac{Z_{be}}{R_{de}}} + j\gamma \right)} \right] \end{aligned} \quad (11.118)$$

where γ is the skewing angle.

A and B are defined in (11.75) and (11.76). Z_{be} and R_{de} are defined in (11.73) and refer to rotor bar impedance and equivalent rotor end ring and bar tooth wall resistance R_{de} is reduced to the rotor bar.

$X_m(f_v)$ is the airgap reactance for frequency f_v and fundamental pole pitch,

$$X_m(f_v) = X_m(f_1) \cdot \frac{f_v}{f_1} \quad (11.119)$$

In [11], for a 500 Hz fundamental, high speed, 2.2 kW motor, the variation of rectangular bar rotor resistance $R_r^*(R_{de})$ for various frequencies was found to have a maximum whose position depends only slightly on frequency (Figure 11.13a).

The rotor leakage reactance $X_{rl}^*(R_{de})$ levels out for all frequencies at high bar-tooth wall contact resistance values (Figure 11.13b).

We should note that Equations (11.115) and (11.116) and Figure 11.13 refer both to skin effect and transverse losses.

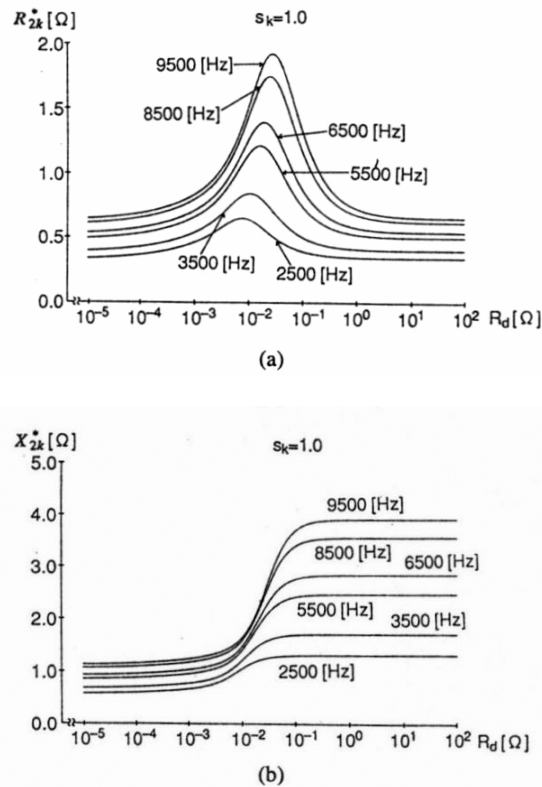


Figure 11.13 Rotor cage resistance and leakage reactance versus bar-tooth wall contact resistance R_{de} , for various time harmonics

As the skin effect at, say, 9500 Hz is very strong and only the upper part of the rotor bar is active, the value of transverse resistance R_{de} is to be increased in the ratio

$$R_{de} = R_{dedc} \frac{h}{\delta_{Al}(f_v)} \quad (11.120)$$

This is as if we would read values of R_r^* , X_{rl}^* in [Figure 11.13](#) calculated for $(R_{de})_{dc}$ at the abscissa R_{de} of (11.120).

Also, as X_{rl}^* increases with R_{de} , the harmonic current and conductor losses tend to decrease for large R_{de} .

Increasing the frequency tends to push the transverse resistance R_{de} to larger values, beyond the critical value and thus lower interbar currents and losses are to be obtained.

However, such a condition has to be verified up to carrier frequency so that a moderate influence of interbar currents is secured.

11.13. COMPUTATION OF TIME HARMONICS CORE LOSSES

The computation of core losses for high frequency in IMs is a very difficult task. However, for the case when $\delta_{Fe} < d$ (large skin effect) and all the flux paths are located around stator and rotor slots and on rotor surface in a thin layer (due to airgap flux pulsation caused by stator slot openings), such an attempt may be made easily through analytical methods.

Core losses may also occur due to axial fluxes, in the stack-end laminations because of end connection high frequency leakage flux. So we have three types of losses here.

- Slot wall time harmonic core losses
- Airgap flux pulsation (zig-zag) time harmonic core losses
- End connection leakage flux time harmonic core losses

11.13.1. Slot wall core losses

Due to the strong skin effect, the stator and rotor currents are crowded toward the slot opening in each conductor layer ([Figure 11.14](#)).

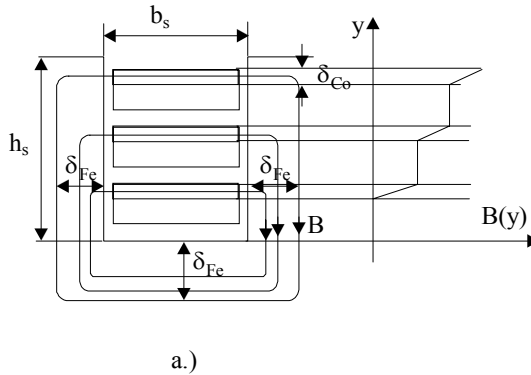


Figure 11.14 Stator a.) and rotor b.) slot leakage flux path and flux density distribution (continued)

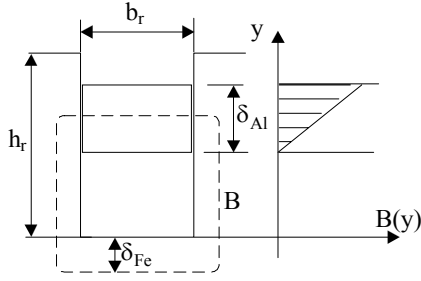


Figure 11.14 (continued)

The stator average leakage flux ϕ_{vs} is

$$\phi_{vs} = \frac{1}{W_1} \left[\frac{L_{s\text{slot}}(f_v)I_v}{\frac{N_s}{3}} \right] \cdot \frac{1}{\left(\frac{h_s}{\xi} \right)} [\text{Wb/m}] \quad (11.121)$$

where W_1 – turns per phase
 $N_s/3$ – slots per stator phase
 h_s/ξ – the total height of m skin depths
 m conductor layers in slot

From [14] the surface core losses, with skin effect considered for the stator, are

$$P_{sw} = \frac{\phi_{vs}^2}{\delta_{Fe}^3 \sigma_{Fe} \mu_{Fe}^2} \cdot \frac{\left(\sinh\left(\frac{2d}{\delta_{Fe}}\right) - \sin\left(\frac{2d}{\delta_{Fe}}\right) \right)}{\left(\cosh\left(\frac{2d}{\delta_{Fe}}\right) - \cos\left(\frac{2d}{\delta_{Fe}}\right) \right)} \cdot K_{er} N_s l_{\text{stack}} (2h_s + b_s) \delta_{Fe} \quad (11.122)$$

K_{er} is the reaction-field factor: $K_{er} = \varphi'$ (see (11.112)) for

$$\xi = \frac{2d}{\delta_{Fe}} \quad (11.123)$$

In a similar way, we may proceed for the rotor slot, but here $m = 1$ (conductors per slot). The rotor average slot leakage flux ϕ_{vr} is

$$\phi_{vr} = \frac{1}{W_2} \left[\frac{L_{r\text{slot}}(f_v)I_v}{\frac{N_r}{m_2}} \right] \cdot \frac{1}{\left(\frac{h_r}{\xi} \right)} [\text{Wb/m}] \quad (11.124)$$

W_2 —turns per rotor phase, m_2 —rotor equivalent phases. For a cage rotor we may consider $W_2 = W_1$ and $m_2 = 3$ as $L_{r\text{slot}}(v)$ is already reduced to the stator.

The rotor slot wall losses P_{rw} are

$$P_{rw} = \frac{\phi_{vr}^2}{\delta_{Fe}^3 \sigma_{Fe} \mu_{Fe}^2} \cdot \frac{\left(\sinh\left(\frac{2d}{\delta_{Fe}}\right) - \sin\left(\frac{2d}{\delta_{Fe}}\right) \right)}{\left(\cosh\left(\frac{2d}{\delta_{Fe}}\right) - \cos\left(\frac{2d}{\delta_{Fe}}\right) \right)} \cdot K_{er} N_r l_{stack} (2h_r + b_r) \delta_{Fe} \quad (11.125)$$

11.13.2. Zig-zag rotor surface losses

The time harmonic airgap flux density pulsation on the rotor tooth heads, similar to zig-zag flux, produces rotor surface losses.

The maximum airgap flux density pulsation B_{vK} , due to stator slot opening is

$$B_{vK} = \frac{\pi}{4} \frac{\beta}{2 - \beta} B_{vg} \quad (11.126)$$

The factor β (b_{os}/g) has been defined in Chapter 5 (Figure 5.4) or Table 10.1. It varies from zero to 0.41 for b_{os}/g from zero to 12. The frequency of B_{vK} is $f_v(N_s/p_1 \pm 1)$. B_{vg} is the f_v frequency airgap space fundamental flux density.

To calculate B_{vg} , we have to consider that all the airgap flux is converted into leakage rotor flux. So they are equal to each other.

$$\left(\frac{2}{\pi} B_{vg} \tau l_{stack} \right) \frac{W_l K_{wl}}{\sqrt{2}} = I_v L_{rv} \quad (11.127)$$

Based on (11.126 and 11.127) in Reference [14], the following result has been obtained for zig-zag rotor surface losses:

$$P_{zr} = 2(\pi D \delta_{Fe}) \frac{1}{8} \frac{\pi^2}{16} \left(\frac{\beta}{2 - \beta} \right)^2 B_{vg}^2 l_{stack}^2 \cdot \frac{K_{er} C_{vIr}}{\delta_{Fe}^3 \sigma_{Fe} \mu_{Fe}^2} \cdot \frac{\left(\sinh\left(\frac{2d}{\delta_{Fe}}\right) - \sin\left(\frac{2d}{\delta_{Fe}}\right) \right)}{\left(\cosh\left(\frac{2d}{\delta_{Fe}}\right) - \cos\left(\frac{2d}{\delta_{Fe}}\right) \right)} \quad (11.128)$$

$$C_{vIr} = \left(\frac{\sin\left(\frac{K\pi K_2}{2R'}\right)}{\left(\frac{K\pi K_2}{2R'}\right)} \right)^2 + \left(\frac{\sin\left(\frac{K\pi K_2}{R'}\right)}{\left(\frac{K\pi K_2}{R'}\right)} \right)^2 - \frac{\sin\left(\frac{K\pi K_2}{2R'}\right)}{\left(\frac{K\pi K_2}{2R'}\right)} \cdot \frac{\sin\left(\frac{K\pi K_2}{R'}\right)}{\left(\frac{K\pi K_2}{R'}\right)} \cdot \cos \frac{K\pi K_2}{2R'} \ll 1 \quad (11.129)$$

$$K = \frac{N_s}{p_1} \pm 1; K_2 = \frac{(t_r - b_{or})}{t_r}; R' = \frac{N_r}{2p_1} \quad (11.130)$$

where t_r —rotor slot pitch; b_{or} —rotor slot opening.

Zig-zag rotor surface losses tend to be negligible, at least in small power motors. End connection leakage core losses proved negligible in small motors. In high power motors, only 3D FEM are able to produce trustworthy results.

In fact, it seems practical to calculate only the slot wall time harmonics core losses in the stator and rotor. For the motor, in Example 11.6 in [14], it was found that $P_{sw} + P_{rw}' = 6.8 + 7.43 = 14.23\text{W}$. This is about 4 times less than conductor losses at 20 kHz.

11.14. LOSS COMPUTATION BY FEM

The FEM, even in its 2D version, allows for the computation of magnetic field distribution once the stator and rotor currents are known. Under no-load, only the stator current waveform fundamental is required. It is thus possible to calculate the additional currents induced in the rotor cage by conjugating field distribution with circuit equations.

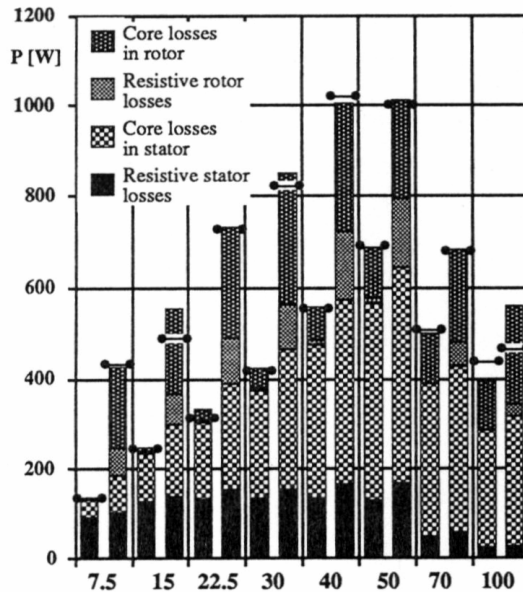


Figure 11.15 No-load losses (37 kW IM) at 7.5 Hz, ..., 100 Hz fundamental frequency.

Left column for sinusoidal voltage and right column for PWM voltage.

Measured losses marked by cross bars [16]

The core loss may be calculated only from the distribution of field in the machine with zero electrical conductivity in the iron core. This is a strong

approximation, especially at high time harmonic frequencies (see previous paragraphs). In References [15, 16] such a computation approach is followed for both sinusoidal and PWM voltages. Sample results of losses for a 37 kW motor on no-load are shown on Figure 11.15. [16]

More important, the flux density radial (B_r) and tangential (B_t) flux density hodographs in 3 points a, b, c (Figure 11.16c) are shown on Figure 11.16a, b. [16] This is proof that the rotor slot opening (point a) and tooth (point b) experience a.c. field while point c (slot bottom) experiences a quasi-traveling field. Although such knowledge is standard, a quantitative proof is presented here.

When the motor is under load, for sinusoidal voltage supply, FEM has been also used to calculate the losses for a skewed bar cage rotor machine. Insulated bars have been used for the computation. This time, again, but justified, the iron skin effect was neglected as time harmonics do not exist. Semiempirical loss formulas, as used with analytical models, are still used with FEM.

The influence of skewing in the rotor is considered separately, and then by using the coupling field-circuit FEM. The stack was sliced into 5 to 8 axial segments properly shifted to consider the skewing effects. [17]

The already documented axial variation of airgap and core flux density due to skewing changes the balance of losses by increasing the stator fundamental and especially the rotor (stray) core losses and decreasing the rotor stray (additional) cage losses.

In low power induction motors, where conductor losses dominate, skewing tends to reduce total losses on load, while for large machines where core loss is relatively more important, the total losses on load tend to increase slightly due to skewing. [17]

The network – field coupled time – stepping finite element 2D model with axial stack segmentation of skewed-rotor cage IMs has been used to include also the interbar currents [18].

The fact that such complex problems can be solved by quasi 2D–FEM today is encouraging as rather reasonable computation times are required, However, all the effects are mixed and no easy way to derive design hints seems in sight.

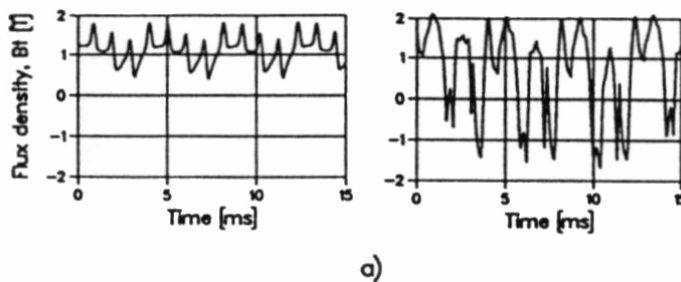


Figure 11.16 No-load flux density hodograph in points a, b, c, at 40 Hz,
left – sinusoidal voltage, right – PWM voltage (continued)

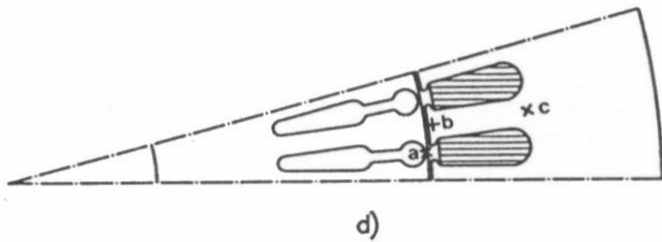
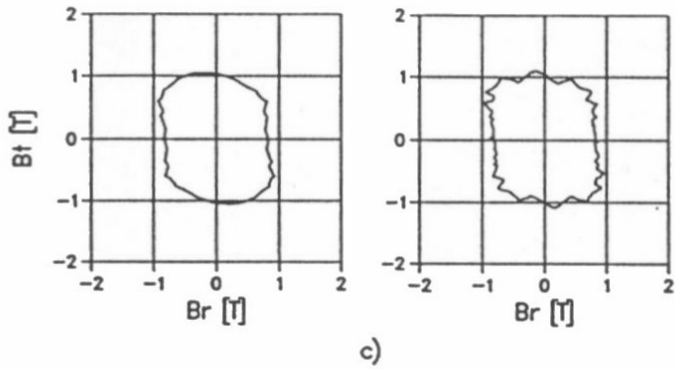
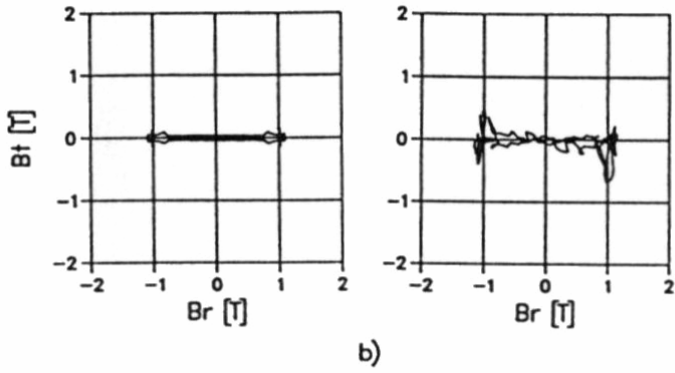


Figure 11.16 (continued)

For other apparently simpler problems, however, as the usage of magnetic wedges for stator slots in large machines, FEM is extremely useful. [19] A notable reduction of rotor stray core losses is obtained. Consequently, higher efficiency for open stator slots is expected.

A similar situation occurs when the hysteresis losses in a cage IM are calculated by investigating the IM from rated to zero positive and negative slip. [20]

A pertinent review of FEM usage in the computation of IM performance (and losses) is given in Reference [21].

11.15. SUMMARY

- Conductor, core, frictional, and windage losses occur in IMs.
- Conductor and core losses constitute electromagnetic losses.
- Electromagnetic losses may be divided into fundamental and harmonic (space and/or time harmonics) losses.
- Fundamental conductor losses depend on skin effect, temperature, and the machine power and specific design.
- Fundamental core losses depend on the airgap flux density, yoke and teeth flux densities, and the supply voltage frequency.
- In a real IM, fundamental electromagnetic losses hardly exist separately.
- Nonfundamental electromagnetic losses are due to space or/and time harmonics.
- The additional (stray) losses, besides the fundamental, are caused by space airgap flux density harmonics or by voltage time harmonics (when PWM converters are used to feed the IM).
- The airgap flux density space harmonics are due to mmf space harmonics, airgap magnetic conductance space harmonics due to slot opening, and leakage or main flux path magnetic saturation.
- No-load stray losses are space harmonics losses at no-load. They are mainly: surface core losses, tooth flux pulsation core losses and tooth flux pulsation cage losses.
- For phase-belt (mmf first) harmonics, 5, 7, 11, 13 and up to the first slot opening harmonic ($N_s/p_1 \pm 1$), the skin depth in laminations is much larger than lamination thickness. So the reaction of core eddy currents on airgap harmonic field is neglected. This way, the rotor (stator) surface core losses are calculated.
- As the number of stator and rotor slots are different from each other $N_s \neq N_r$, the stator and rotor tooth flux pulsates with N_r and N_s pole pairs, respectively, per revolution. Consequently, tooth flux pulsation core losses occur both in the stator and rotor teeth.
However, such tooth flux pulsations and losses are attenuated by the corresponding currents induced in the rotor cage.
- For the rotor slot skewing by one stator slot pitch, the reaction of cage currents to rotor tooth flux pulsation is almost zero and thus large rotor tooth flux pulsation core losses persist (undamped).
- For straight slot rotors, the rotor tooth flux pulsation produces, as stated above, no-load tooth flux pulsation cage losses. They are not to be neglected, especially in large power motors.

- All space harmonics losses under load are called stray load losses.
- Under load, the no-load stray losses are “amplified”. A load amplification factor C_{load} is defined and used to correct the stator and rotor tooth flux pulsation core losses. A distinct load correction factor is used for calculating rotor tooth flux pulsation cage losses. For full pitch stator winding when calculating stray losses, besides the first slot harmonics $N_s \pm p_1$, the first phase belt harmonics (5, 7) are contributing to losses. In skewed rotors, the load stray cage losses are also corrected by a special factor.
- Cast aluminum cages are used with low to medium power induction machines. Their bars are noninsulated from slot walls and thus a transverse (cross-path) resistance between bars through iron occurs. In this case, interbar current losses occur especially for skewed rotors. Thus, despite of the fact that with proper skewing, the stray cage losses are reduced, notable interbar current losses may occur making the skewing less effective.
- There is a critical transverse resistance for which the interbar current losses are maximum. This condition is to be avoided by proper design. Interbar current losses occur both on no-load and on load.
- A number of rules to reduce stray losses are presented. The most interesting is $N_s > N_r$ where even straight rotor slots may be used after safe starting is secured.
- With the advent of power electronics, supply voltage time harmonics occur. With PWM static power converters, around carrier frequency, the highest harmonics occur unless random PWM is used. Frequencies up to 20 kHz occur this way.
- Both stator and rotor conductor losses, due to these voltage time harmonics, are heavily influenced by the skin effect (frequency). In general, as the frequency rises over 2 to 3kHz, for given time harmonic voltage, the conductor losses decrease slightly with frequency while core losses increase with frequency.
- The computation of core losses at high time frequencies (up to 20 kHz) is made accounting for the skin depth in iron δ_{Fe} as all field occurs around slots and on the rotor surface in a thin layer (δ_{Fe} – thick). These slot wall and rotor surface core losses are calculated. Only slot wall core losses are not negligible and they represent about 20 to 30% of all time harmonic losses.

FEM has been applied recently to calculate all no-load or load losses, thus including implicitly the space harmonics losses. Still the core losses are determined with analytical expressions where the local flux densities variation in time is considered. For field distribution computation, the laminated core electrical conductivity is considered zero. So the computation of time harmonic high frequencies (20 kHz) core losses including the iron skin depth is not available yet with FEM. The errors vary from 5 to 30%.

- However, the effect of skewing, interbar currents, magnetic wedges, and relative number of slots N_s/N_r has been successfully investigated by field-coupled circuit 2D FEMs for reasonable amounts of computation time.
- New progress with 3D FEM is expected in the near future.
- Measurements of losses will be dealt with separately in the chapter dedicated to IM testing.

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